California State University – Los Angeles
Department of Mathematics and Computer Science
Syllabus for the
Master's Degree Comprehensive Examination

Fall 1984
(Retyped June 1995, Hoffman)
(Retyped April 2000, Hoffman)

Topics

1. Algebra and geometry of complex numbers

2. Mapping properties of elementary functions. (exponential, powers, trigonometric functions, linear fractional (Möbius) transformations, branching of logarithm and roots)

3. Analyticity and the Cauchy-Riemann equations.

4. Cauchy theorem and the Cauchy integral formulas

5. Relationship to harmonic functions: Harmonic conjugates, Dirichlet problem

6. Understand and apply convergence theory for analytic and harmonic functions.

7. Series expansions of Taylor and Laurent (manipulation, convergence, regions of convergence, etc.)

8. Classification and behavior of singularities.

9. Residues and applications to definite integrals, including improper real integrals, and series

10. Theoretical considerations:

 (1) Casorati-Weierstrass theorem
 (2) Cauchy estimates
 (3) Liouville's theorem about entire functions
 (4) Maximum modulus principle
 (5) Schwarz' lemma
 (6) Identity theorem and analytic continuation
 (7) Morera's theorem
 (8) "Principle of the argument", root counting formula, Rouche's theorem

11. Conformal mapping: elementary results, Möbius transformations, some applications to the Dirichlet problem, statement of the Riemann mapping theorem.

References

[**1**]   R. V. Churchill, J. Brown, and R. Verhey, *Complex Variables amd Applications*, McGraw-Hill Book Co.

[**2**]   James B. Conway, *Functions of One Complex Variable* , 2nd ed. Graduate Texts in Mathematics no. 11, Springer-Verlag New York

[**3**]   Konrad Knopp, *Elements of the Theory of Functions*, Dover Publications Inc.

[**4**]   _____ , *Theory of Functions: Part I*, Dover Publications Inc.

[**5**]   _____ , *Theory of Functions: Part II*, Dover Publications Inc.

[**6**]   _____ , *Problem Book in the Theory of Functions: Vol. I*, Dover Publications Inc.

[**7**]   _____ , *Problem Book in the Theory of Functions: Vol. II*, Dover Publications Inc.

[**8**]   A. I. Markushevich, *Theory of Functions*, 3 volumes in 1, Chelsea Publishing Co. (Vol. 1, Vol. 2 sections 1, 2, 5)

[**9**]   Jerrold Marsden and Michael Hoffman, *Basic Complex Analysis*, 3rd ed., W. H. Freeman and Co., 1999

[**10**]   Murray R. Spiegel, *Complex Variables*, Schaum's Outlines, McGraw-Hill Book Co.