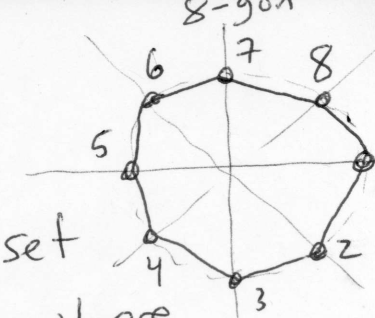
Dihedral groups



For each $n \geq 3$, let D_{2n} be the set of symmetries of a regular n-gon, where a symmetry is any rigid motion of the n-gon which can be effected by taking a copy of the n-gon, moving this copy in any fashion in 3-space and then placing the copy back on the original n-gon so it exactly covers it. The identity symmetry fixes ~~does nothing to~~ the n-gon. Given a symmetry σ, σ^{-1} is the symmetry that "undoes" σ. ~~The group operation is composition of symmetries.~~

Given two symmetries $\sigma_1, \sigma_2 \in D_{2n}$ the group operation $\sigma_1 \sigma_2$ means first apply σ_2 to the n-gon and then apply σ_1 to the n-gon.

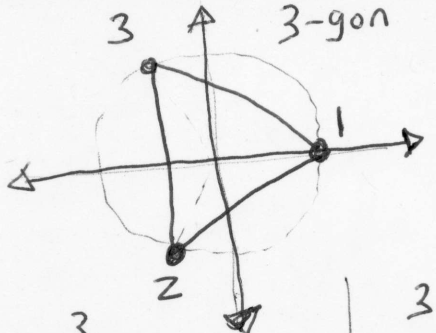
$\sigma_1 \sigma_2$

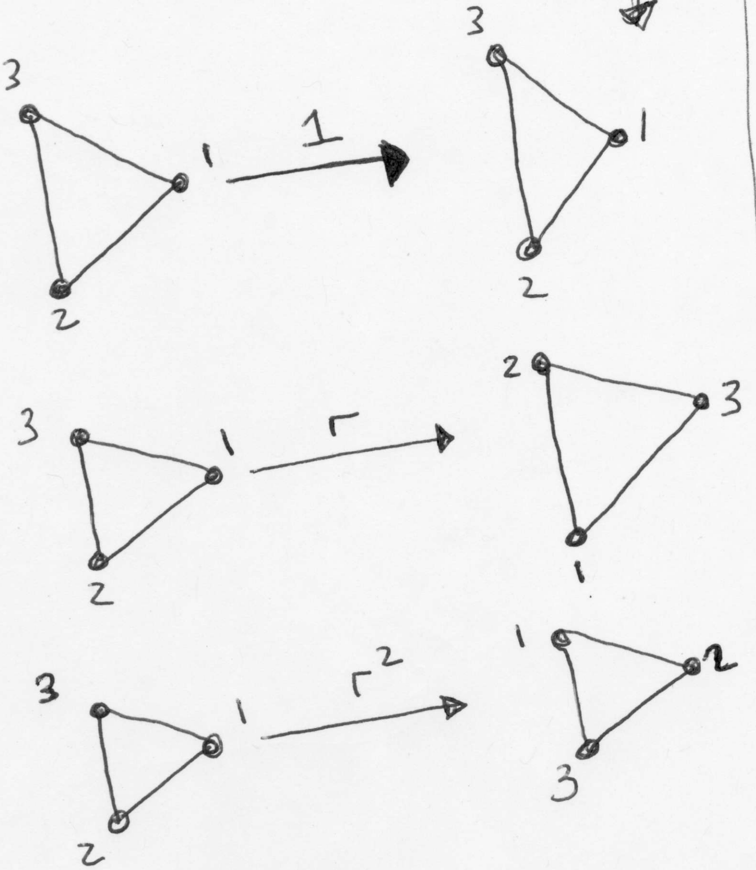
$\longleftarrow$ read this way when doing group op.

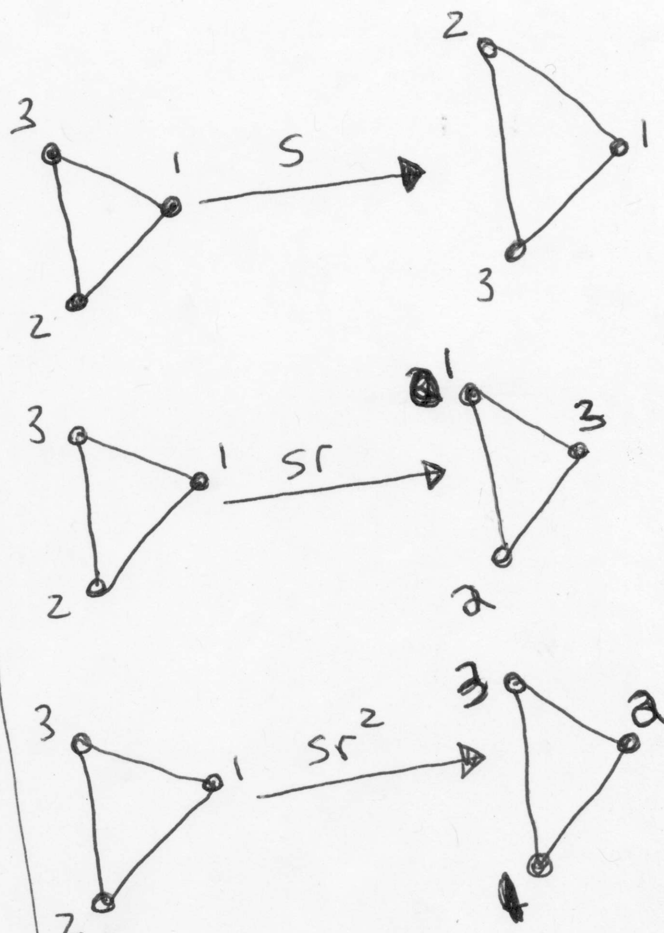
Fact: D_{2n} is a group using this group operation.

Ex: D_6

(Do 1, r, s first, then do the others)

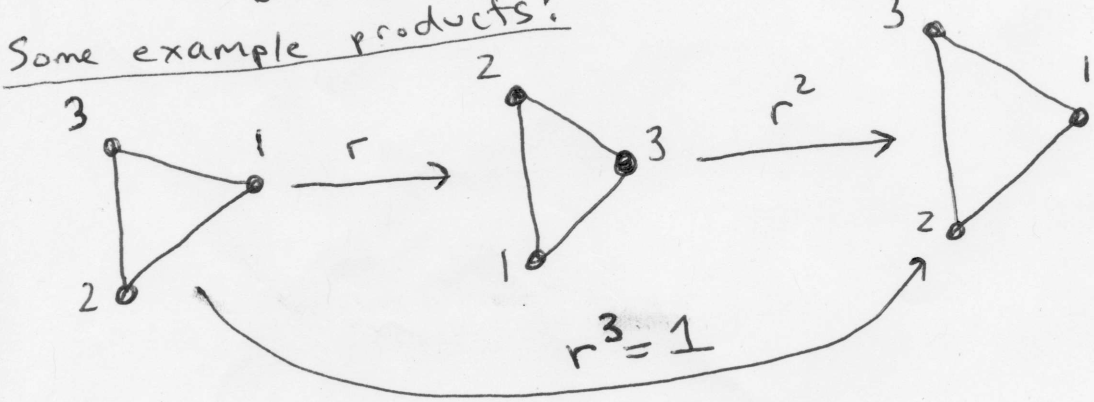




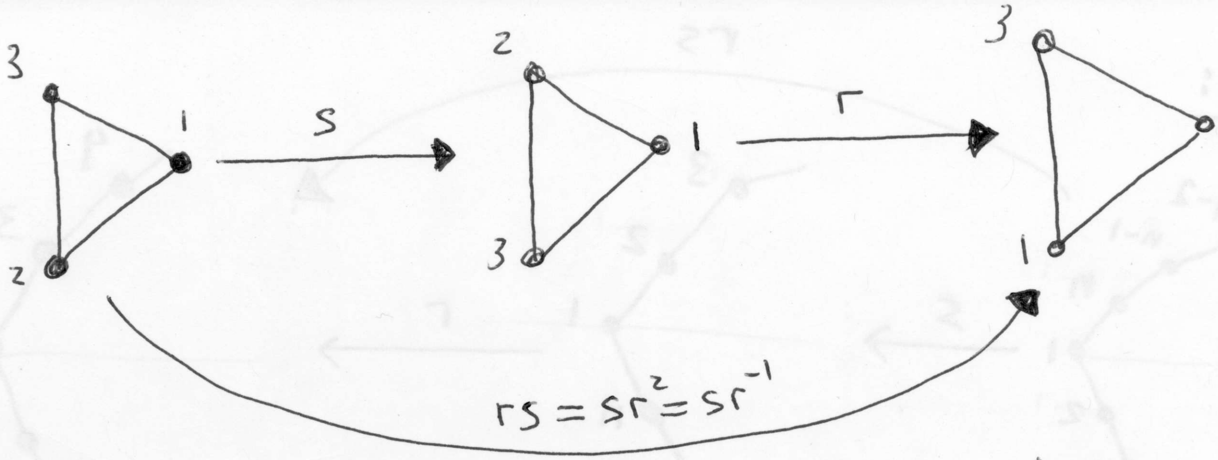


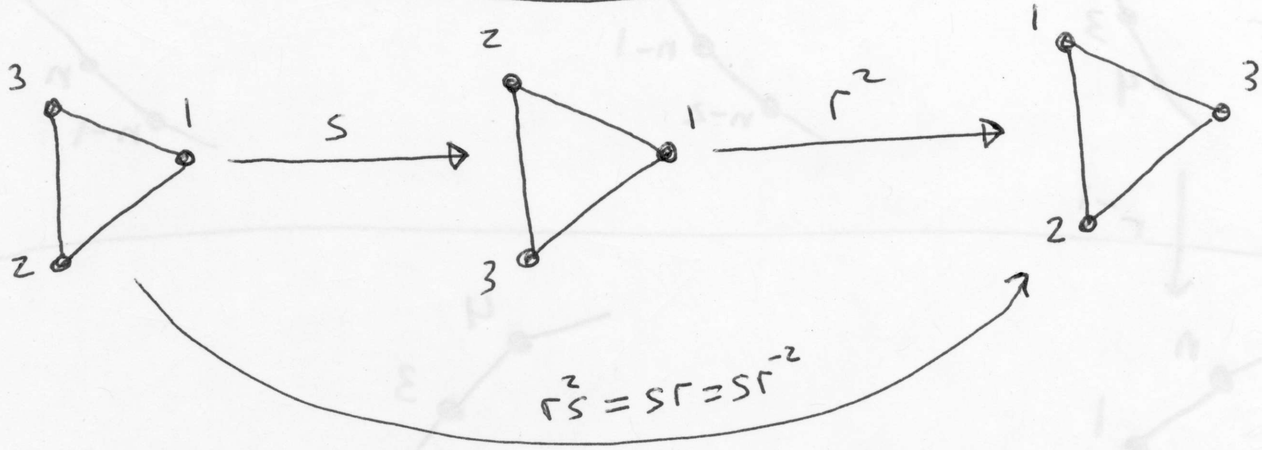
$$D_6 = \{1, r, r^2, s, sr, sr^2\}$$

Some example products:



So, $r^{-1} = r^2$.

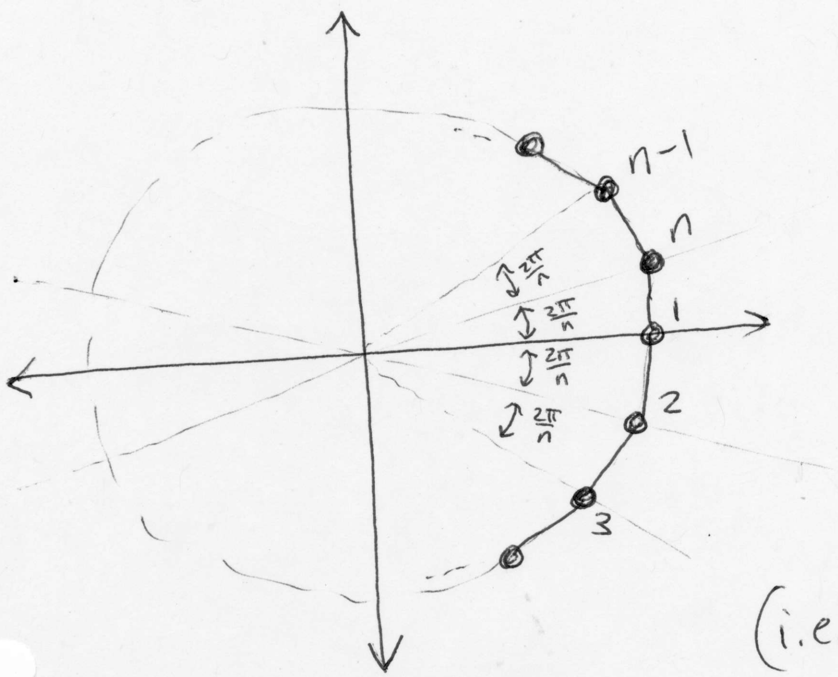




- D_0 D_6 table (go forward 2 pages to see it)

D_{2n} in general

Fix an n-gon centered in the origin in the xy-plane and label the vertices consecutively from 1 to n in a clockwise manner.



Let r be the rotation clockwise about the origin through $\frac{2\pi}{n}$ radians.

Let s be the reflection about the line of symmetry through vertex 1 and the origin

(i.e., s flips across the x-axis)

Then:

1. $1, r, r^2, \ldots, r^{n-1}$ are all distinct and $r^n = 1$, Also, $r^{-1} = r^{n-1}$.
2. $s^2 = 1$. So, $s^{-1} = s$.
3. $s \neq r^i$ for any i
4. $sr^i \neq sr^j$ for $0 \leq i < j \leq n-1$
5. $D_{2n} = \{1, r, r^2, \ldots, r^{n-1}, s, sr, sr^2, \ldots, sr^{n-1}\}$
6. $rs = sr^{-1}$
7. $r^i s = s\underbrace{r^{-i}}_{\text{where } r^{-i} \text{ means } (r^{-1})^i}$ for $0 \leq i \leq n$.

This shows how to commute s past powers of r.

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~~(iii) Since $a*(a^{-1}) = (a^{-1})*a = e$ by the definition of inverse, $(a^{-1})^{-1} = a$.~~

~~(iv) $(a*b)*(b^{-1}*a^{-1}) = e \Rightarrow b^{-1}*a^{-1} = (a*b)^{-1}$.~~

~~Def: Let G be a group and $x \in G$. If there is a positive integer m such that~~

D_6	1	r	r^2	s	sr	sr^2
1	1	r	r^2	s	sr	sr^2
r	r	r^2	1	sr^2	s	sr
r^2	r^2	1	r	sr	sr^2	s
s	s	sr	sr^2	1	r	r^2
sr	sr	sr^2	s	r^2	1	r
sr^2	sr^2	s	sr	r	r^2	1

$sr^2sr = r^{-2}r = r^{-1} = r^2$
$(sr^2)(sr^2) = 1$

$rs = sr^{-1} = sr^2$
$rsr = sr^{-1}r = s$
$rsr^2 = sr^{-1}r^2 = sr$

$r^2s = sr^{-2} = s(r^2)^2 = sr^4 = sr$
$r^2sr = sr^{-2}r = sr^{-1} = sr^2$
$r^2sr^2 = sr^{-2}r^2 = s$

$srs = r^{-1} = r^2$
$(sr)(sr) = 1$
$(sr)(sr^2) = r$