HW 3 # 1(g)

Show that $\lim_{x \to 2} (x^3 - 1) = 7$

* $f(x) = x^3 - 1$ has domain $D = \mathbb{R}$

proof: Let $\varepsilon > 0$

Need to show: Need to find $\delta > 0$ where if

$0 < |x-2| < \delta$ then $|(x^3-1)-7| < \varepsilon$

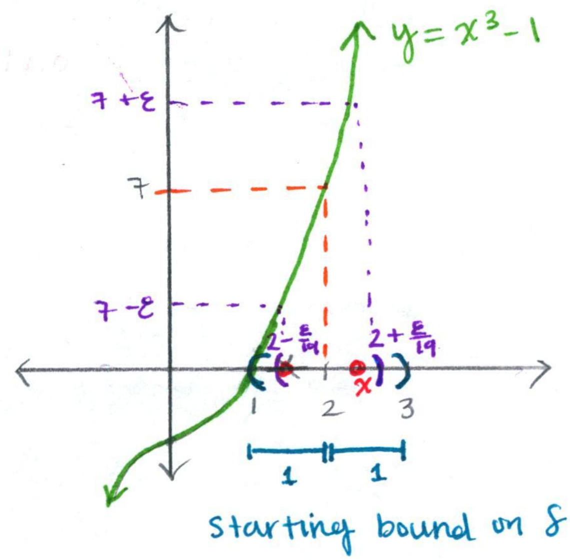
Note to ourselves; we can leave out $x \in D$ since $D = \mathbb{R}$ is all x.

Note that,

$$|(x^3-1)-7| = |x^3-8| = \underbrace{|x-2|}_{\text{we can control this \# with } \delta}\ \underbrace{|x^2+2x+4|}_{\text{we want to bound this part by a \#}}$$

$$\begin{array}{r|l} & x^2+2x+4 \\ \hline x-2 & x^3-8 \\ & -(x^3-2x^2) \\ \hline & 2x^2-8 \\ & -(2x^2-4x) \\ \hline & 4x-8 \\ & -(4x-8) \\ \hline & 0 \end{array}$$

Suppose $\delta \leq 1$



Suppose $\delta \leq 1$ ← This # is arbitrarily picked

Supposed $0 < |x-2| < \delta \leq 1$

Then $-1 < x-2 < 1$

i.e. $1 < x < 3$

So if $0 < |x-2| < 1$ then $|x| < 3$

So if $0 < |x-2| < 1$ then $|(x^3-1)-7| = |x-2||x^2+2x+4|$

$\leq |x-2|(|x^2| + |2x| + |4|) = |x-2|(|x|^2 + 2|x| + 4)$

$\underset{|x|<3}{<} |x-2|(9 + 2 \cdot 3 + 4) = 19|x-2|$

Let $\delta = \min\{1, \frac{\varepsilon}{19}\}$

If $0 < |x-2| < \delta$ then $\Bigg\}$ This implies that $0<|x-2|<1$ and $0<|x-2|<\frac{\varepsilon}{19}$

$$|(x^3-1)-7| = |x-2|\,|x^2+2x+4| \underset{0<|x-2|<1}{\leq} 19\cdot|x-2| \underset{0<|x-2|<\frac{\varepsilon}{19}}{<} 19\cdot\frac{\varepsilon}{19} = \varepsilon \quad \boxtimes$$

Continuity (HW 4)

Def: Let $D \subseteq \mathbb{R}$ and $f: D \to \mathbb{R}$ and $a \in D$.

(1) We say that f is **continuous at a** if for every $\varepsilon > 0$ there exists $\delta > 0$ such that if $|x-a| < \delta$ and $x \in D$ then $|f(x) - f(a)| < \varepsilon$

(2) If $B \subseteq D$ and f is continuous at all $b \in B$ then we say that f is **continuous on B**.

Note: Let $a \in D$

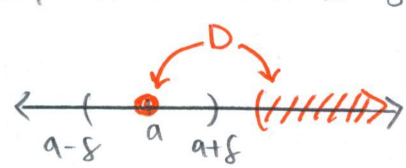
case 1: If a is a limit point on D in part (1) of the def, then f is continuous at a iff

(1) $f(a)$ is defined

(2) $\lim_{x \to a} f(x)$ exists

(3) $\lim_{x\to a} f(x) = f(a)$

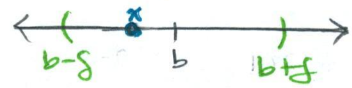
case 2: If a is not a limit point of D, then there exists $\delta > 0$ where $(a-\delta, a+\delta) \cap D = \{a\}$.



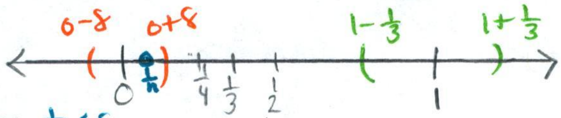
Thus given $\varepsilon > 0$, pick $\delta > 0$ to be as above. Then if $|x-a| < \delta$, we have that $x = a$. Then $|f(x) - f(a)| = |f(a) - f(a)| = 0 < \varepsilon$ so f is continuous at a.

P.2 10/10

b is a limit point of D if for every $\delta > 0$ there exists $x \in D$ with $x \neq b$ and $|x-b| < \delta$



Ex: $D = \{ \frac{1}{n} \mid n = 1, 2, 3, \dots \}$



1 is not a limit point of D

where $\frac{1}{n} < \delta$
0 is a limit pt.

Ex:

$$f(x) = \begin{cases} x^2, & x \le 0 \\ 3, & x = 5 \\ \text{undefined}, & x > 0 \text{ and } x \neq 5 \end{cases}$$