2.1

1(b) $G = \{ z \in \mathbb{C} \mid |z| = 1 \}$.

G is a subset of $\mathbb{C} \setminus \{0\}$ which is a group under multiplication. $G \neq \emptyset$ since $|1| = 1$ and so $1 \in G$. Let $x, y \in G$.

Then $|x| = 1$ and $|y| = 1$.

So, $\left| \frac{x}{y} \right| = \frac{|x|}{|y|} = \frac{1}{1} = 1$.

Thus, $\frac{x}{y} \in G$. By Prop 1, page 47, G is a subgroup of $\mathbb{C} \setminus \{0\}$.

2(a)

$(1,2), (2,3)$ are 2-cycles in S_n if $n \geq 3$. But $(1,2)(2,3) = (1,2,3)$ is not a 2-cycle.

2.1

3(b) $H = \{1, r^2, sr, sr^3\}$

	1	r^2	sr	sr^3
1	1	r^2	sr	sr^3
r^2	r^2	1	sr^3	sr
sr	sr	sr^3	1	r^2
sr^3	sr^3	sr	r^2	1

Computations:

$(sr)(sr^3) = ssr^{-1}r^3$
$= r^2$

$(sr^3)(sr) = s^2 r^{-3} r$
$= r^{-2} = r^2$

The table shows that H is closed under the group operation. Also, we see that $(r^2)^{-1} = r^2 \in H$, $(sr)^{-1} = sr \in H$ and $(sr^3)^{-1} = sr^3 \in H$. And $1 \in H$.

So, $H \leq D_8$.

$\boxed{2.1}$

10(a)

Since $H \leq G$, we have that $1 \in H$.
Since $K \leq G$, we have that $1 \in K$.
Hence $H \cap K \neq \phi$.
Let $x, y \in H \cap K$.
Since $H \leq G$, we have that $y^{-1} \in H$ and $xy^{-1} \in H$.
Since $K \leq G$, we have that $y^{-1} \in K$ and $xy^{-1} \in K$.

Hence $xy^{-1} \in H \cap K$.
By Prop 1, page 47, $H \cap K \leq G$.

12(b)

Let $H = \{a \in A \mid a^n = 1\}$.

Since $1^n = 1$ we see that $1 \in H$.

Thus $H \neq \phi$. Let $x, y \in H$.

Then $x^n = 1$ and $y^n = 1$. Since $y^n = 1$ we have that $(y^n)^{-1} = 1^{-1} = 1$ and so $(y^{-1})^n = 1$.

Thus, $(xy^{-1})^n = x^n (y^{-1})^n = 1 \cdot 1 = 1$.

since A is abelian

So, $xy^{-1} \in H$. Thus, $H \leq A$.

Let $H = \{ x \in D_{2n} \mid x^2 = 1 \}$ where $n \geq 3$.

(14) $s^2 = 1$ and $(sr)^2 = 1$.

Thus, $s \in H$ and $sr \in H$.

But $r^2 \neq 1$ if $n \geq 3$. So, $r \in H$.

However, $s \cdot (sr) = r$. So, H is not closed under the group operation of D_{2n} so it is not a subgroup of D_{2n}.