3.2

4. Suppose that $|G| = pq$ where p and q are primes. Let $Z(G)$ be the center of G. By Lagrange, $|Z(G)| = 1, p, q,$ or pq.

<u>case 1: $|Z(G)| = pq$</u> In this case $G = Z(G)$ and so G is abelian.

<u>case 2: $|Z(G)| = p$</u> Then $|G/Z(G)| = q$. Since q is prime, $G/Z(G)$ is cyclic. By 3.1 #36, G is abelian.

<u>case 3: $|Z(G)| = q$</u> This is the same as the $|Z(G)| = p$ case.

<u>case 4: $|Z(G)| = 1$</u> This is one of the outcomes in the statement of the problem.

So, in summary, either G is abelian or $Z(G) = \{1\}$.

3.2

(8) Suppose that $|H| = h$ and $|K| = k$ and $\gcd(h,k) = 1$.

Let $n = |H \cap K|$. Since, $H \cap K \leq H$, by Lagrange, n divides h.

Since $H \cap K \leq K$, by Lagrange, n divides k.

So, $n = 1$ since $\gcd(h,k) = 1$.

So, $H \cap K = \{1\}$.

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(16) We did this one in class.