Math 446 - Homework # 1

In the following problems, x, y, z, m, n are integers.

1. Prove that if $x|y$ and $y|z$, then $x|z$.

2. Prove that if $x|y$ and $m|n$, then $xm|yn$.

3. Prove that if $xy|z$, then $x|z$.

4. Prove that $xz|yz$ if and only if $x|y$.

5. Prove that if $x|(y+z)$ and $x|y$, then $x|z$.

6. Prove that if $x|y$ and $x|z$, then $x|(my+nz)$.

7. Let $n > 1$ be an integer.

   (a) n is composite if and only if there exist positive integers a and b such that $n = ab$ and $1 < a < n$ and $1 < b < n$.

   (b) n is composite if and only if there exist positive integers a and b such that $n = ab$ and $1 < a$ and $1 < b$.

8. Prove that 4 does not divide $n^2 + 2$ for any integer n.

9. Prove that any prime of the form $3k + 1$ is of the form $6s + 1$.

10. Show that $n^4 + 4$ is composite for all $n > 1$.

11. Let $n > 1$ be an integer. If $2^n - 1$ is a prime, then n is prime. [An integer of the form $2^p - 1$, where p is prime is called a Mersenne prime.]

12. If $d|n$ and $d|n+1$, then $d = 1$ or $d = -1$.