Math 5800

Homework # 2

4650 review

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1. Let $(b_n)_{n=1}^{\infty}$ be a non-decreasing sequence of real numbers that converges to L. Prove that $b_n \leq L$ for all $n \geq 1$.

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2. Let $(a_n)_{n=1}^{\infty}$ be a non-increasing sequence of real numbers with $\lim\limits_{n\to\infty} a_n = L$. Prove that $a_n \geq L$ for all $n \geq 1$.

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3. Given $a, b \in \mathbb{R}$ define $\max\{a,b\}$ and $\min\{a,b\}$ as follows:

$$\max\{a,b\} = \begin{cases} a & \text{if } b \leq a \\ b & \text{if } a < b \end{cases} \qquad \text{and} \qquad \min\{a,b\} = \begin{cases} a & \text{if } a \leq b \\ b & \text{if } b < a \end{cases}$$

Let $s, t \in \mathbb{R}$ and let $(s_n)_{n=1}^{\infty}$ and $(t_n)_{n=1}^{\infty}$ be sequences of real numbers. Suppose that s_n converges to s and t_n converges to t.

(a) Prove that the sequence $(\max\{s_n, t_n\})_{n=1}^{\infty}$ converges to $\max\{s,t\}$.

(b) Prove that the sequence $(\min\{s_n, t_n\})_{n=1}^{\infty}$ converges to $\min\{s,t\}$.

[The above problem is from Weir page 7, problem 2. Here's a hint: Given $\epsilon > 0$ there exists $N > 0$ such that

$$s - \epsilon < s_n < s + \epsilon \quad \textit{and} \quad t - \epsilon < t_n < t + \epsilon$$

for $n \geq N$. From this it follows that

$$max\{s,t\} - \epsilon < max\{s_n, t_n\} < max\{s,t\} + \epsilon$$

for $n \geq N$. Similarly for min.]

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4. Let $(a_n)_{n=1}^{\infty}$ be a sequence of real numbers. Prove that if (a_n) is a convergent sequence, then it is a bounded sequence.

[To prove the above statement, follow these steps:

(a) Let L be the limit of (a_n). Pick $\epsilon = 1$. Show that there exists an $N > 0$ where

$$|a_n| \leq |L| + 1$$

for all $n \geq N$.

(b) Show that $|a_n| \leq max\{|a_1|, |a_2|, \ldots, |a_{N-1}|, |L| + 1\}$ for all $n \geq 1$.]