Math 5680

Homework # 4 - Part 2

More on Laurent series and residues

1. Each function f below has an isolated singularity the point z_0 that is given. For each problem do the following: (i) Classify the singularity as removable, a pole of order k, or an essential singularity, and (ii) find the residue of f at z_0.

   (a) $f(z) = \dfrac{e^z - 1}{\sin(z)}$, at $z_0 = 0$

   (b) $f(z) = \dfrac{1}{e^z - 1}$, at $z_0 = 0$

   (c) $f(z) = \dfrac{z + 2}{z^2 - 2z}$, at $z_0 = 0$

   (d) $f(z) = \dfrac{e^z}{(z^2 - 1)^2}$, at $z_0 = 1$

   (e) $f(z) = \dfrac{e^{z^2}}{(z - 1)^4}$, at $z_0 = 1$

   (f) $f(z) = \dfrac{z^2}{z^4 - 1}$, at $z_0 = i$

   (g) $f(z) = \left(\dfrac{\cos(z) - 1}{z}\right)^2$, at $z_0 = 0$

2. Find all the singular points and residues of $f(z) = \dfrac{1}{e^z - 1}$.

3. Find all the singular points and residues of $f(z) = \dfrac{1}{z^3 - 3}$.

4. Suppose that f_1 and f_2 both have simple poles at z_0. Prove that $f_1 \cdot f_2$ has a pole of order 2 at z_0 and find a formula for the residue at z_0.