Math 2550
HW 4 - Part 2
Inverses

1. Given that the system

$$\begin{array}{rcrcrcl} 2x_1 & - & 4x_2 & + & 5x_3 & = & 4 \\ -x_1 & & & + & x_3 & = & 2 \\ x_1 & - & 4x_2 & + & 6x_3 & = & 7 \end{array}$$

   has no solution, determine whether or not the matrix

$$A = \begin{pmatrix} 2 & -4 & 5 \\ -1 & 0 & 1 \\ 1 & -4 & 6 \end{pmatrix}$$

   is invertible. Explain why your answer is correct.

2. Suppose that A, B, P, Q are all $n \times n$ matrices.

   Suppose that $B^2 = I$.

   Suppose that $A = PBQ$ and that P and Q are inverses.

   Prove that $A^2 = I$

3. Let A be a 3×3 matrix.

   Let O be the 3×3 zero matrix.

   Let I be the 3×3 identity matrix.

   Suppose that $A^3 = O$.

   Prove that $I - A$ is invertible and that $(I - A)^{-1} = I + A + A^2$.

4. Let A, C, D be $n \times n$ matrices.

   Let I be the $n \times n$ identity matrix.

   Suppose that $CA = I$ and $AD = I$.

   Prove that $C = D$.

5. Suppose that A is an $n \times n$ matrix.

   Let $\vec{y}$ and $\vec{z}$ be in $\mathbb{R}^n$.

   Suppose that $\vec{y} \neq \vec{z}$.

   Suppose that $A\vec{y} = A\vec{z}$.

   Prove that A is not invertible.

6. Suppose that A is an $n \times n$ matrix and $\vec{b}$ is in $\mathbb{R}^n$.

   Suppose that the equation $A\vec{x} = \vec{b}$ has infinitely many solutions for $\vec{x}$.

   Does A^{-1} exist?