

Math 4680 - Homework # 5
Analytic functions and derivatives

1. Where are the following functions analytic? Give a formula for $f'(z)$ where $f'(z)$ exists.
 - (a) $13z^7 - 3z^4 + 1$
 - (b) $\frac{3z^2 - 1}{2 - z}$
 - (c) $\frac{\cos(z)}{\sin(z)}$
 - (d) $\left(\frac{1}{z - 1}\right)^{100}$
 - (e) 5^z , defined using the principal branch of the logarithm
 - (f) $\log(z + 1)$, where $\log(u)$ is the principal branch of the logarithm
 - (g) $z^{(1+i)}$, defined using the principal branch of the logarithm
 - (h) $\sqrt{z - 2}$, defined using the principal branch of the logarithm
2. Show that $f'(z)$ does not exist at any point if
 - (a) $f(z) = |z|$
 - (b) $f(z) = e^{\bar{z}}$
3. Determine where $f'(z)$ exists and find its formula for $f'(z)$ where it exists. Where is $f(z)$ analytic?
 - (a) $f(x + iy) = x^2 + iy^2$
 - (b) $f(z) = z \cdot \text{Im}(z)$
4. Let
$$f(z) = \begin{cases} (\bar{z})^2/z & \text{when } z \neq 0 \\ 0 & \text{when } z = 0 \end{cases}$$
Show that $f'(0)$ does not exist.
5. Let g be analytic on an open set A . Let $B = \{z \in A \mid g(z) \neq 0\}$. Show that (i) B is open and (ii) $1/g$ is analytic on B .