

Math 5680

Homework # 5

The residue theorem

1. Calculate the following integrals

(a) $\int_{|z|=1/2} \frac{dz}{(1-z)^3}$ where the curve is oriented counter-clockwise

(b) $\int_{\gamma} \frac{dz}{(1-z)^3}$ where γ is the circle centered at 1 with radius 1, oriented counter-clockwise

(c) $\int_{|z-2|=2} \frac{e^z}{(1-z)^3} dz$ where the curve is oriented counter-clockwise

(d) $\int_{|z-1|=1/2} \frac{e^z}{z(1-z)^3} dz$ where the curve is oriented counter-clockwise

(e) $\int_{\gamma} \frac{e^z}{z^2(z-1)^3} dz$ where γ is the circle centered at 0 with radius 2, oriented counter-clockwise

2. (a) Show that $\cos(z) = 0$ if and only if $z = \frac{\pi}{2} + \pi n$ where $n \in \mathbb{Z}$.

(b) Evaluate $\int_{\gamma} \frac{\sin(z)}{\cos(z)} dz$ where γ is a circle of radius π centered at π .

3. Evaluate $\int_{\gamma} \frac{1}{e^z - 1} dz$ where γ is a circle of radius 9 centered at 0.

4. Evaluate $\int_{\gamma} \frac{e^{z^2}}{z^2} dz$ where γ is the square with the four vertices $1 + i$, $-1 + i$, $-1 - i$, and $1 - i$.