Math 456

Homework # 8 - Irreducible polynomials and the ring
$$R/I = F[x]/\langle p(x)\rangle$$

1. Let F be a field. Let $c \in F$. Then $x - c$ is a divisor of $f(x)$ in $F[x]$ if and only if $f(c) = 0$.

2. Determine which polynomials are irreducible over $\mathbb{Z}_n[x]$. If one is reducible, keep factoring it until you can't factor any more.

   (a) $x^2 + \bar{1}$ in $\mathbb{Z}_3[x]$.

   (b) $x^2 + \bar{2}$ in $\mathbb{Z}_3[x]$.

   (c) $x^2 + x + \bar{1}$ in $\mathbb{Z}_3[x]$.

   (d) $x^4 + \bar{4}$ in $\mathbb{Z}_5[x]$.

3. Determine whether $x^5 - 5x^3 + 195x + 10$ is irreducible in $\mathbb{Q}[x]$.

4. Determine whether $x^2 - 2$ is irreducible in $\mathbb{Q}[x]$.

5. Determine whether $x^{10} - 10$ is irreducible in $\mathbb{Q}[x]$.

6. Find a finite field of size 4. List out the elements and how you constructed the field.

7. Find a finite field of size 8. List out the elements and how you constructed the field.