

## Math 455

### Homework # 9 - Factor groups and normal subgroups

1. Let  $G = \mathbb{Z}_{12}$  and  $H = \langle \bar{4} \rangle$ . Calculate the elements of  $G/H$ . Find the order of  $\bar{5} + H$  in  $G/H$ . Find the order of  $\bar{6} + H$  in  $G/H$ .
2. Let  $G = \mathbb{Z}_4 \times \mathbb{Z}_4$  and  $H = \langle (\bar{1}, \bar{1}) \rangle$ . Calculate the elements of  $G/H$ . Find the order of  $(\bar{3}, \bar{1}) + H$  in  $G/H$ . Find the order of  $(\bar{2}, \bar{3}) + H$  in  $G/H$ .
3. Let  $G = \mathbb{Z}_2 \times \mathbb{Z}_4$  and  $H = \langle (\bar{0}, \bar{1}) \rangle$ . Find a familiar group  $G'$  that  $G/H$  is isomorphic to and use the first isomorphism theorem to prove it.
4. Let  $G = \mathbb{Z}_2 \times \mathbb{Z}_4$  and  $H = \langle (\bar{0}, \bar{2}) \rangle$ . Find a familiar group  $G'$  that  $G/H$  is isomorphic to and prove it.
5. Let  $G = \mathbb{Z} \times \mathbb{Z}$  and  $H = \langle (1, 1) \rangle$ . Find a familiar group  $G'$  that  $G/H$  is isomorphic to and use the first isomorphism theorem to prove it.
6. Let  $G$  be a finite group and  $H$  be a subgroup of  $G$ . Prove that if  $H$  is only subgroup of  $G$  of size  $|H|$ , then  $H$  is normal in  $G$ .
7. Let  $G$  be a group and  $H$  and  $K$  be normal subgroups of  $G$ . Prove that  $H \cap K$  is a normal subgroup of  $G$ .
8. Let  $\phi : G \rightarrow G'$  be an onto homomorphism and let  $N$  be a normal subgroup of  $G$ . Prove that  $\phi(N)$  is a normal subgroup of  $G'$ .
9. Let  $\phi : G \rightarrow G'$  be an homomorphism and let  $N'$  be a normal subgroup of  $G'$ . Prove that  $\phi^{-1}(N')$  is a normal subgroup of  $G$ .