7

Since $\lim_{n\to\infty} a_n = A$ and $\lim_{n\to\infty} b_n = B$

we know that $\lim_{n\to\infty}(a_n - b_n) = A - B$.

Let $\varepsilon > 0$.

Then there exists $N > 0$ where if $n \geq N$ then $|(a_n - b_n) - (A-B)| < \varepsilon$

Pick some fixed $m \geq N$.

Then $|a_m - b_m - A + B| < \varepsilon$.

Thus, $-\varepsilon < a_m - b_m - A + B < \varepsilon$

Recall $|x| < c$ means $-c < x < c$ if $c > 0$

Thus,

$$-\varepsilon - a_m + b_m - B < -A < \varepsilon - a_m + b_m - B$$

In particular,

$$-\varepsilon - a_m + b_m - B < -A.$$

Multiplying by -1 gives

$$A < \varepsilon + (a_m - b_m) + B$$

Recall that $a_m \leq b_m$.

Thus, $a_m - b_m \leq 0$.

So, we get that

$$A < \varepsilon + \underbrace{(a_m - b_m)}_{\leq 0} + B \leq \varepsilon + B$$

So, $A < \varepsilon + B$.

Thus, $A - B < \varepsilon$.

Summarizing, we have that $A - B < \varepsilon$ for every $\varepsilon > 0$.

Hence $A - B \leq 0$.

Thus, $A \leq B$.