USING THE TI-89 TO CONVEY MATHEMATICAL CONCEPTS: AN INTRODUCTORY MODELING COURSE FOR NON-SCIENCE MAJORS

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1   Introduction

This workshop provides a hands-on introduction to a modeling course designed with a Course and Curriculum Development grant from the National Science Foundation[1]. The unique feature of this course is its intended audience: Non-science majors, especially students who take only one mathematics course during their undergraduate program. For most of these students mathematics applications consist of balancing checkbooks and computing percentages. In the new modeling course, students gain an understanding of the use of mathematical models across many disciplines and learn how models can be used to predict the future or to decide on policy.

The course focuses on model development, test of model against data and interpretation of results. Quantitative and qualitative interpretations of long-term model behavior and sensitivity to model parameters are derived for both discrete and continuous models. The goal is to have students experience the modeling process and to open their eyes to the fact that their everyday lives are influenced by decisions based on mathematical models.

In the past, a modeling course would not have been possible at a freshman/sophomore level, as the calculus sequence and a differential equations course are common pre-requisites for modeling courses. However, the availability of computer algebra systems (CAS) on graphing calculators such as the TI-89 and TI-92 can be utilized to perform the more advanced computations, allowing students without the relevant mathematics background to explore and analyze mathematical models.

Since the target student audience for the modeling course has only a limited mathematical background, the models developed in this course are very basic, and many theoretical results are derived heuristically, using a combination of numerical and graphical approach. However, unlike in the more advanced theoretical courses, comparison of the model predictions to real data is an integral part of the course. This connection between the theoretical models and their application engages the students, making the course content relevant to their lives, and increases their appreciation of mathematics. With an understanding of the modeling process, in particular the fact that models need to be validated by data and that model predictions may be limited, students will be able to

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critically evaluate conflicting information on topics such as Global Warming and to put model predictions into the proper context.

2 History of Course Development

Different teaching experiences over the years have lead to the development of this course, which was originally based on Mathematica. At Humboldt State University, I taught a modeling course that required the calculus sequence, but not the differential equations course as a prerequisite. To give students a better understanding of the differential equations and also to show how models with linear and non-linear differential equations differ, the differential equations package MacMath (developed by Hubbard and West) was used to display vector fields for differential equations in one and two variables. It was amazing how this experience changed the students' perception. The most dramatic difference took place for students who did not have a very strong mathematical background - they were asking when they could go to the lab again!

Later on, at California State University Los Angeles (CSLA) I was regularly teaching one section of the General Educations mathematics course, and was thinking about the content of that course. It seemed that a more beneficial course for this particular student audience could be designed, oriented toward modeling, in the spirit of the calculus and precalculus reform movement. With the advent of Mathematica version 3.0, and in particular the feature of user-defined palettes, the idea for the modeling course was born. Use of palettes would allow me to built a copy-and-paste user interface that would avoid the usual beginner's frustrations (capitalization, misspelling of function names). While the course was designed, CAS became available on graphing calculators such as the TI-89 and the TI-92. Even though the screen real estate of the TI-89 is limited, it has the advantage of mobility – at a commuter campus such as CSLA, that is a major issue! As a result, a second version of the course based on the TI-89 was developed.

3 Incorporation of Technology and Pedagogy

Use of technology, whether computers or graphing calculators, introduces an additional level of difficulty for students in any course. However, more and more students have used graphing calculators in high school, and the percentage of students not familiar with a graphing calculator is bound to decrease over time. Furthermore, the additional functions developed by the author follow the conventions of the built-in functions, and are introduced with examples in special calculator lessons.

A second, major concern in any course that relies to a great amount on technology is the creation of a black-box approach, where the computer or calculator is the source of all answers blindly accepted by the students. To avoid this problem, students are performing computations initially using paper and pencil (whenever suitable) to strengthen their conceptual understanding. In addition, students are required to make predictions about the model results before solving the model using the calculator. Model results are discussed and interpreted in the context of the problem both orally and in writing.

Student assignments consist of exercises reinforcing new definitions or methods, as well as open-ended questions that require both computations and writing. These exercises extend models presented in class by either asking additional questions based on the model or by introducing variations to the model. A final class project guides students through the complete modeling process: Formulating a model, testing it against data, discussing possible modifications and justifying their final choice for the model. These projects are presented to the class either orally or as a poster session.

4 Course Content

The course starts by describing the sequence of models for the solar system proposed over the centuries. This illustrates the iterative nature of the modeling process, specifically the fact that new data may indicate that a model needs to be revised. The remainder of the course is divided into three principal parts: Part I defines the concept of a function and leads to a model derivation by means of a least squares fit to a given set of data. Part II derives discrete models (single and multiple variables) from assumptions, while part III focuses on continuous models.

Using the input-output paradigm, the concept of a function is defined, and the various representations (list of values, graph, or formula) for functions are presented. Polynomials (of degree ≤ 3), exponential, logistic and sine functions are introduced and graphical and numerical characteristics for each are discussed. These properties are then used to decide which type of function should be fitted to given real-world data using the method of least squares fit. The idea behind the least squares method is illustrated with an example of a linear fit. Once students have identified likely function types for a given data set, they use the calculator to perform the least squares fit for those function types. Comparing the respective errors and a look at the context of the problem leads to a final model, which is then used for predictions. Figure 1 shows data on the median age of first marriage for women in the US, and the best model, and a logistic fit.
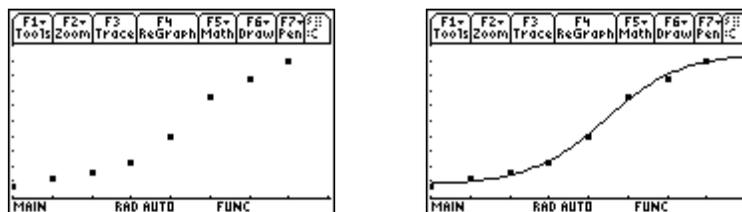

Figure 1: Fitting Functions to Data

In the second part of the course, discrete models (single- and multi-variable) are derived from assumptions using the paradigm **new = old + change**. The resulting iterative model equation is used to analyze the behavior of the model. Specific questions that cannot be answered precisely using the iterative model equation are used to motivate the search for an explicit solution. In addition, the notion of equilibrium is introduced via examples. A brief excursion into classification of discrete dynamical systems (linearity and order) follows, then theorems for the explicit solution and equilibrium values (of a single

variable linear DDS) are stated. For non-linear models, Cobweb diagrams are introduced, and stability of equilibria and the long-term behavior of the model are discussed using tables of model values and their graphs. Examples cover exponential models for population growth and finance, logistic models for improved population models and to model the spread of a disease, as well as simple genetic models. The latter models progress from the simplest model of asexual reproduction to explain genetic drift to models that include sexual reproduction and mutation.

For linear systems of several equations, students first are introduced to matrix notation as a way to capture the essence of the system. Again, through examples, students discover an explicit solution in terms of powers of the system matrix. Furthermore, they are introduced to the notions of Eigenvalues and Eigenvectors as means to predict the long-term behavior of the system. Non-linear models are presented as well, and are evaluated using the iterative model equation. Models with several variables include population models which incorporate age structure, simple probabilistic models and predator-prey models (non-linear). Figure 2 shows the long-term behavior of a model for two similar species which compete for the same food source.
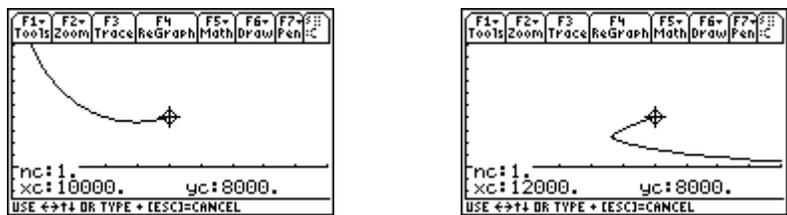

Figure 2: Model Behavior for Different Initial Values

The last part of the course makes the connection between discrete and continuous models. The latter type of models are based on the rate of change of a quantity. Using the example of average and instantaneous velocity, the interpretation of the derivative as rate of change is introduced. Side by side with the numerical derivation, the graphical interpretation of the derivative as the limit of secant slopes is developed and visualized through an animation. The two notions of the derivative are combined for the qualitative analysis of a differential equation. Using the slope field, students graphically derive the model solution and discuss the sensitivity of the long-term behavior to initial conditions. Examples used include exponential decay and the continuous version of systems like the predator-prey model. All models are restricted to one or two variables, as these are the only ones that can be displayed visually using vector fields. Figure 3 shows the vector field for a differential equation of one variable, and solutions for different initial values.
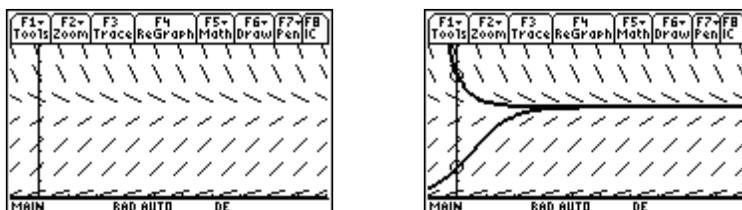
Figure 3: Qualitative Analysis for Continuous Models

5 Student Response

The course (in the Mathematica version) has been taught three times at CSLA. Generally, student reaction was very positive. Students expressed the fact that they could "see" mathematics and that they had a newfound appreciation of its use. Here are some comments from the course evaluations:

"This class has been a great ego booster in regard to my math inadequacies. I usually have no concept of math and for once I have been understanding it"

"Most of my experiences with math courses have always been somewhat uninteresting and frustrating. [Using] *Mathematica however, is a new approach that is still very challenging but adds a new element - it is interesting and you really get to see how math works outside the classroom. I'll recommend the course to other students"*

"I think this is a good course, because it uses examples that can actually be applied in our days. For example, making predictions on certain topics, it also gives a good taste of computer knowledge. I really enjoyed this course."

6 Course Material

The first part of the lecture notes (for both versions), the special Mathematica and TI-89 functions and introductory lessons on the use of the respective technology tool are posted at the course web site (www.cinenet.net/~silvi), which can also be accessed via link from the author's home page (www.calstatela.edu/faculty/sheubac). Additional lecture notes will be posted as they are being transcribed from handwritten notes into typeset form. Please send any feedback on the lecture notes and supplements to the author at sheubac@calstatela.edu.