Lemma: Let G be a group. Suppose $x \in G$ and x has order n. If $x^k = 1$ for some integer k, then n divides k.

proof: By the division algorithm $k = qn + r$ where $0 \leq r < n$ for some $q, r \in \mathbb{Z}$.

Also,

$$1 = x^k = x^{qn+r} = (x^n)^q x^r = 1 \cdot x^r = x^r.$$

Since $0 \leq r < n$ and x has order n and $x^r = 1$, we must have $r = 0$. Thus, $k = qn$.

So, n divides k. ▨