

Syllabus for Examination in Applied Linear Analysis

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This examination involves topics from linear algebra, linear spaces, analysis, differential equations, and integral equations commonly found in the courses MATH 215, 255, 457, 465, 502A, and 502B.

TOPICS

I. Vector spaces (finite and infinite dimensional)

- (1) Norms and inner products
- (2) Bases, orthonormal bases, orthogonalization
- (3) Projections and approximation
- (4) Convergence of sequences and series of vectors
- (5) Fourier series and generalized Fourier series. Bessel and Parseval relations
- (6) Contraction mapping theorem

II: Linear functionals and linear operators.

- (1) Matrix representation for a linear transformation
- (2) Boundedness, continuity and the norm of an operator
- (3) Linear functionals and dual spaces
- (4) Eigenvalues and eigenvectors
- (5) Adjoints
- (6) Diagonalization
- (7) Compact operators

III: Integral Equations

- (1) Integral operators
- (2) Fredholm equations and Volterra equations
- (3) Separable kernels, symmetric kernels, convolution operators
- (4) Solution methods including successive approximation, resolvent kernel, Neumann series, eigenfunction expansion, Fourier series
- (5) Correspondence of integral equations with differential equations
- (6) Existence and Uniqueness of solutions

IV: Differential equations

- (1) Conversion between differential and integral equations
- (2) Sturm-Liouville systems
- (3) Green's functions
- (4) Eigenfunction expansions
- (5) Existence and uniqueness of solutions.

REFERENCES

Here is a list, in no particular order, of some books which have been used for the courses or which people have found useful. The list is long since everybody has their favorites.

- [1] G. F. Roach, *Green's Functions* 2nd ed. , Cambridge Univ. Press, 1982
- [2] F. G. Tricomi, *Integral Equations*, Dover Publications, 1985 (Chapters 1 and 2)
- [3] N. Young, *An Introduction to Hilbert Space*, Cambridge Univ. Press, 1988.
- [4] H. Hochstadt, *Integral Equations*, John Wiley & Sons, 1988.
- [5] L. Debnath and P. Mikusinski, *Introduction to Hilbert Space with Applications*, Academic Press, 1990.
- [6] G. Birkhoff and G.-C. Rota, *Ordinary Differential Equations*, 3rd ed. John Wiley & Sons, 1978. (Chapters 10,11)
- [7] F. B. Hildebrandt, *Methods of Applied Mathematics*, 2nd ed. Dover Publications, 1965 (1992).
- [8] E. Kreyszig, *Introduction to Functional Analysis with Applications*, John Wiley & Sons.
- [9] A. Jerri, *Introduction to Integral Equations with Applications*, Marcel Dekker.
- [10] R. Courant and D. Hilbert, *Methods of Mathematical Physics*, Vol. 1, John Wiley & Sons.