Weds
1/29

Continued from last time.

Ex: $\displaystyle\int_0^{\frac{3\sqrt{3}}{2}} \frac{x^3}{(4x^2+9)^{3/2}}\,dx$

$= \circ\circ\circ\circ = \dfrac{3}{16}\displaystyle\int_0^{\pi/3} \frac{\sin^3(\theta)}{\cos^2(\theta)}\,d\theta$

a bunch
of stuff
happened here

$= \dfrac{3}{16}\displaystyle\int_0^{\pi/3} \frac{\sin\ldots}{\cos\ldots}$

$= \dfrac{3}{16}\displaystyle\int_0^{\pi/3} \ldots$

$= -\dfrac{3}{16}\displaystyle\int_1^{1/2} \ldots$

$u=\cos(\theta)$
$du=-\sin(\theta)d\theta$
$-du=\sin(\theta)d\theta$

me.

$$\ldots\frac{\ldots^3(\theta)}{\ldots s^2(\theta)}\,d\theta$$

$$\frac{3}{16}\int_0^{\pi/3} \underbrace{\frac{\sin^2(\theta)}{\cos^2(\theta)}}_{\text{turn into cosines}} \underbrace{\sin(\theta)\,d\theta}_{\text{save for } du}$$

$$= \frac{3}{16}\int_0^{\pi/3} \frac{1-\cos^2(\theta)}{\cos^2(\theta)} \sin(\theta)\,d\theta$$

$$= -\frac{3}{16}\int_1^{1/2} \frac{1-u^2}{u^2}\,du$$

$u=\cos(\theta)$	When $\theta=\frac{\pi}{3}$, then $u=\cos\left(\frac{\pi}{3}\right)=\frac{1}{2}$
$du=-\sin(\theta)\,d\theta$	
$-du=\sin(\theta)\,d\theta$	When $\theta=0$, then $u=\cos(0)=1$

$$-\frac{3}{16}\int_1^{1/2}\left(\frac{1}{u^2}-\frac{u^2}{u^2}\right)du$$

$$= -\frac{3}{16}\int_1^{1/2}\left(u^{-2}-1\right)du$$

$$= -\frac{3}{16}\left[\frac{u^{-1}}{-1}-u\right]_1^{1/2}$$

$$= -\frac{3}{16}\left[\left(-\frac{1}{\left(\frac{1}{2}\right)}-\frac{1}{2}\right)-\left(-1-1\right)\right]$$

$$-\frac{3}{16}\left(-\frac{5}{2}+2\right)$$

$$=\left(-\frac{3}{16}\right)\left(-\frac{1}{2}\right)$$

$$=\boxed{\frac{3}{32}}$$

7.5 — Partial Fractions

In this section we will consider integrals of the form $\int \frac{P(x)}{Q(x)}\,dx$ where $P(x)$ and $Q(x)$ are polynomials.

First look at $\frac{P(x)}{Q(x)}$.

If the degree of $P(x)$ is greater than or equal to the degree of $Q(x)$, then first divide $Q(x)$ into $P(x)$ to simplify.

Ex: Consider $\displaystyle\int \frac{x^3+x}{x-1}\,dx$

degree 3

degree 1

Since $3 \geq 1$ we simplify by division of polys:

$$
\begin{array}{r|l}
 & x^2 + x + 2 \\
\hline
x-1 & x^3 + x \\
 & -(x^3 - x^2) \\
\hline
 & x^2 + x \\
 & -(x^2 - x) \\
\hline
 & 2x \\
 & -(2x-2) \\
\hline
 & 2
\end{array}
$$

$$x^3+x = (x^2+x+2)(x-1)+2$$

$$\frac{x^3+x}{x-1} = (x^2+x+2) + \frac{2}{x-1}$$

So, $\int \frac{x^3+x}{x-1}\,dx = \int \left[(x^2+x+2) + \frac{2}{x-1}\right] dx$

$$= \boxed{\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\ln|x-1| + C}$$

Useful integral

$$\int \frac{dx}{ax+b} = \frac{1}{a}\cdot\ln|ax+b| + C$$

Here a & b & C are constants

Next step is to factor $Q(x)$.

Depending on how $Q(x)$ factors there are different techniques you can use to integrate.

Case 1: Suppose $Q(x)$ factors into distinct linear factors. That is,

$$\frac{P(x)}{Q(x)} = \frac{P(x)}{(a_1x+b_1)(a_2x+b_2)\cdots(a_nx+b_n)}$$

where $(a_1x_1+b_1), (a_2x+b_2), \ldots, (a_nx+b_n)$ are distinct, ie not the same.

Then it's possible to find constants $A_1, A_2, \ldots, A_n$ where

$$\frac{P(x)}{Q(x)} = \frac{A_1}{a_1x+b_1} + \frac{A_2}{a_2x+b_2} + \cdots + \frac{A_n}{a_nx+b_n}$$

Then integrate each term.

We

7.

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We'll continue this next time.

7.3

$\boxed{\#19}$ $\int \cos^3(x)\sqrt{\sin(x)}\,dx = \int \cos^2(x)\sqrt{\sin(x)}$

$= \int (1-\sin^2(x))\sqrt{\sin(x)}\,\cos(x)dx \underset{\uparrow}{=} \int (1-u^2)\sqrt{u}\,du = \int (1-u^2)u^{1/2}\,du$

$\boxed{\begin{aligned} u &= \sin(x) \\ du &= \cos(x)\,dx \end{aligned}}$

$= \int (u^{1/2} - u^{5/2})\,du = \frac{u^{3/2}}{3/2} - \frac{u^{7/2}}{7/2} + C$

$= \boxed{\frac{2}{3}\sin^{3/2}(x) - \frac{2}{7}\sin^{7/2}(x) + C}$