

Weds  
1/29

Continued from last time.

Ex:  $\int_0^{\frac{3\sqrt{3}}{2}} \frac{x^3}{(4x^2+9)^{3/2}} dx$

$= \dots = \frac{3}{16} \int_0^{\pi/3} \frac{\sin^3(\theta)}{\cos^2(\theta)} d\theta$

a bunch  
of stuff  
happened here

$\frac{3}{16} \int_0^{\pi/3} \frac{\sin^3(\theta)}{\cos^2(\theta)} d\theta$   
 $= \frac{3}{16} \int_0^{\pi/3} \frac{1}{\cos^2(\theta)} d\theta$   
 $= -\frac{3}{16} \int_1^{1/2} \frac{1}{u} du$   
 $u = \cos(\theta)$   
 $du = -\sin(\theta) d\theta$   
 $-du = \sin(\theta) d\theta$

me

$$\frac{3}{16} \int_0^{\pi/3} \frac{\sin^2(\theta)}{\cos^2(\theta)} \sin(\theta) d\theta$$

+ turn into cosines  
 save for du

$$= \frac{3}{16} \int_0^{\pi/3} \frac{1 - \cos^2(\theta)}{\cos^2(\theta)} \sin(\theta) d\theta$$

$$= -\frac{3}{16} \int_1^{1/2} \frac{1-u^2}{u^2} du$$

$u = \cos(\theta)$ $du = -\sin(\theta) d\theta$ $-du = \sin(\theta) d\theta$	when $\theta = \frac{\pi}{3}$ , then $u = \cos(\frac{\pi}{3}) = \frac{1}{2}$
	when $\theta = 0$ , then $u = \cos(0) = 1$

$$= -\frac{3}{16} \int_1^{1/2} \left( \frac{1}{u^2} - \frac{u^2}{u^2} \right) du$$

$$= -\frac{3}{16} \int_1^{1/2} (u^{-2} - 1) du$$

$$= -\frac{3}{16} \left[ \frac{u^{-1}}{-1} - u \right]_1^{1/2}$$

$$= -\frac{3}{16} \left[ \left( -\frac{1}{\frac{1}{2}} - \frac{1}{2} \right) - (-1 - 1) \right] = \frac{3}{32}$$

$\frac{\sin^2(\theta)}{\cos^2(\theta)} d\theta$

## 7.5 — Partial Fractions

In this section we will consider integrals of the form

$$\int \frac{P(x)}{Q(x)} dx$$

where  $P(x)$  and  $Q(x)$  are polynomials

First look at  $\frac{P(x)}{Q(x)}$ .

If the degree of  $P(x)$  is greater than or equal to the degree of  $Q(x)$ , then first divide  $Q(x)$  into  $P(x)$  to simplify.

Ex:

Consider

$$\int \frac{x^3 + x}{x-1} dx$$

degree  
3

degree  
1

Since  $3 \geq 1$  we simplify by division of polys.

$$\begin{array}{r} x-1 \overline{) \begin{array}{r} x^3 + x + 2 \\ \underline{-(x^3 - x^2)} \\ x^2 + x + 2 \\ \underline{-(x^2 - x)} \\ 2x + 2 \\ \underline{-(2x - 2)} \\ 2 \end{array}} \end{array}$$

$$x^3 + x = (x^2 + x + 2)(x-1) + 2$$

$$\frac{x^3 + x}{x-1} = (x^2 + x + 2) + \frac{2}{x-1}$$

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**EVACUATION**  
Evacuation is the process of leaving a building or area in a safe and orderly manner. It is a critical part of emergency preparedness. Always follow the instructions of the fire warden or other designated personnel. Do not use elevators. Do not return to the building until you are told to do so.

**SHELTER IN PLACE**  
Shelter in place is a procedure that is used to protect people from a hazard that is outside the building. It is used when the hazard is not immediately life-threatening and the building is not damaged. Stay in the building and follow the instructions of the fire warden or other designated personnel.

**POWER OUTAGE**  
A power outage is a loss of electrical power. It can be caused by a variety of factors, including weather, equipment failure, or human error. If you experience a power outage, stay calm and follow the instructions of the fire warden or other designated personnel.

**FIRE**  
Fire is a major hazard in any building. It can spread quickly and cause significant damage and injury. If you see a fire, call the fire department immediately and follow the instructions of the fire warden or other designated personnel.

$$\text{So, } \int \frac{x^3 + x}{x-1} dx = \int \left[ (x^2 + x + 2) + \frac{2}{x-1} \right] dx$$

Useful integral

$$\int \frac{dx}{ax+b} = \frac{1}{a} \cdot \ln|ax+b| + C$$

Here  $a$  &  $b$  &  $C$   
are constants

$$= \frac{x^3}{3} + \frac{x^2}{2} + 2x + 2 \ln|x-1| + C$$

Next step is to factor  $Q(x)$ . Depending on how  $Q(x)$  factors there are different techniques you can use to integrate.

Case 1: Suppose  $Q(x)$  factors into distinct linear factors. That is,

$$\frac{P(x)}{Q(x)} = \frac{P(x)}{(a_1x+b_1)(a_2x+b_2)\dots(a_nx+b_n)}$$

where  $(a_1x+b_1), (a_2x+b_2), \dots, (a_nx+b_n)$  are distinct, i.e. not the same.

Then it's possible to  
find constants  $A_1, A_2, \dots, A_n$   
where

$$\frac{P(x)}{Q(x)} = \frac{A_1}{a_1x+b_1} + \frac{A_2}{a_2x+b_2} + \dots + \frac{A_n}{a_nx+b_n}$$

Then integrate each term.

We

7.

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We'll continue this next time.

7.3

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$$\int \cos^3(x) \sqrt{\sin(x)} dx = \int \cos^2(x) \sqrt{\sin(x)}$$

$$= \int (1 - \sin^2(x)) \sqrt{\sin(x)} \cos(x) dx$$

$$\begin{array}{l} u = \sin(x) \\ du = \cos(x) dx \end{array}$$

$$\int (1 - u^2) \sqrt{u} du = \int (1 - u^2) u^{1/2} du$$

$$= \int (u^{1/2} - u^{5/2}) du = \frac{u^{3/2}}{3/2} - \frac{u^{7/2}}{7/2} + C$$

$$= \frac{2}{3} \sin^{3/2}(x) - \frac{2}{7} \sin^{7/2}(x) + C$$