1/30 Thursday

Ex: Evaluate $\int \frac{x^2+2x-1}{2x^3+3x^2-2x}$

$\frac{x^2+2x-1}{2x^3+3x^2-2x}$ $\leftarrow$ degree 2, $\leftarrow$ degree 3

Since top degree < bottom degree no need to divide denominator into numerator

So we factor the denominator as much as possible.

$\frac{x}{2\ldots}$

The ...
So ...

$\frac{x^2+\ldots}{x(2x\ldots}$

degree
degree
nominator
erator

$$\frac{x^2+2x-1}{2x^3+3x^2-2x} = \frac{x^2+2x-1}{x(2x^2+3x-2)} = \frac{x^2+2x-1}{x(2x-1)(x+2)}$$

There are no repeated factors in the denominator so we can find constants A, B, C where

$$\frac{x^2+2x-1}{x(2x-1)(x+2)} = \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{x+2}$$

How do we find A, B, C ?

Step 1 Multiply through by $x(2x-1)(x+2)$.

We get

$$(*) \quad \boxed{x^2+2x-1 = A(2x-1)(x+2) + Bx(x+2) + Cx(2x-1)}$$

Method 1 to solve (*) for A, B, C

$$x^2+2x-1 = A(2x^2+3x-2) + B(x^2+2x) + C(2x^2-x)$$

$$x^2+2x-1 = (2A+B+2C)x^2 + (3A+2B-C)x + (-2A)$$

$1 = 2A+B+2C$

$2 = 3A+2B-C$

$-1 = -2A$

$$\boxed{\begin{aligned} 2A+B+2C &= 1 \\ 3A+2B-C &= 2 \\ -2A &= -1 \end{aligned}} \xleftarrow{} \boxed{A=\tfrac{1}{2}}$$

(plug in $A=\frac{1}{2}$)

$$\boxed{\begin{aligned} B+2C &= 0 \\ 2B-C &= \tfrac{1}{2} \end{aligned}} \leftarrow B=-2C$$

$-5C = \frac{1}{2} \rightarrow \boxed{C = -\frac{1}{10}}$

$\boxed{B = -2C = \frac{1}{5}}$

$\boxed{A=\frac{1}{2},\ B=\frac{1}{5},\ C=-\frac{1}{10}}$

Method 2 to solve (*) for A, B, C

(*) $x^2+2x-1 = A(2x-1)(x+2)+Bx(x+2)+Cx(2x-1)$

Plug in $x=0$: $-1 = A(-1)(2)+B(0)+C(0)$

$\boxed{A=\frac{1}{2}}$

Plug in $x=\frac{1}{2}$: $\left(\frac{1}{2}\right)^2+2\left(\frac{1}{2}\right)-1 = A(0)+B\left(\frac{1}{2}\right)\left(\frac{1}{2}+2\right)+C(0)$

$\frac{1}{4} = B\left(\frac{5}{4}\right) \longrightarrow \boxed{B=\frac{1}{5}}$

x-1)

c(0)

Plug in $x=-2$

$$(-2)^2+2(-2)-1 = A(0)+B(0)+C(-2)(2(-2)-1)$$

$$-1 = 10C$$

$$\boxed{C=-\frac{1}{10}}$$

So, $\boxed{A=\frac{1}{2},\ B=\frac{1}{5},\ C=-\frac{1}{10}}$

Back in time...

$$\int \frac{x^2+2x-1}{2x^3+3x^2-2x}\,dx = \int \left[\frac{A}{x} + \frac{B}{2x-1} + \frac{C}{x+2}\right] dx$$

$$= \int \left(\frac{\left(\frac{1}{2}\right)}{x} + \frac{\left(\frac{1}{5}\right)}{2x-1} + \frac{\left(\frac{-1}{10}\right)}{x+2}\right) dx$$

$$\frac{1}{2}\int\frac{dx}{x}+\frac{1}{5}\int\frac{dx}{2x-1}-\frac{1}{10}\int\frac{dx}{x+2}$$

$$=\frac{1}{2}\ln|x|+\frac{1}{5}\cdot\frac{1}{2}\ln|2x-1|-\frac{1}{10}\ln|x+2|+C$$

$$=\boxed{\frac{1}{2}\ln|x|+\frac{1}{10}\ln|2x-1|-\frac{1}{10}\ln|x+2|+C}$$

$$\boxed{\int\frac{dx}{ax+b}=\frac{1}{a}\ln|ax+b|+C}$$

$$= \ln\left(|x|^{1/2}\right) + \ln\left(|2x-1|^{1/10}\right) + \ln\left(|x+2|^{-1/10}\right) + C$$

$$= \ln\left(|x|^{1/2}\,|2x-1|^{1/10}\,|x+2|^{-1/10}\right) + C$$

$$\boxed{\ln(a)+\ln(b) = \ln(ab)}$$

$$= \ln\left(\frac{|x|^{1/2}\,|2x-1|^{1/10}}{|x+2|^{1/10}}\right) + C$$

The book does this. You don't need to.

7.2

37 $\displaystyle\int_{1/2}^{\sqrt{3}/2} \underbrace{\sin^{-1}(y)}_{u}\, \underbrace{dy}_{dv}$

$= \displaystyle y\sin^{-1}(y)\Big|_{1/2}^{\sqrt{3}/2} - \int_{1/2}^{\sqrt{3}/2} \frac{y}{\sqrt{1-y^2}}\,dy =$

$u = \sin^{-1}(y) \qquad du = \dfrac{1}{\sqrt{1-y^2}}\,dy$

$dv = dy \qquad v = y$

LIATE

u ⟶

7.2

7

$$= \left(\frac{\sqrt{3}}{2}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)\right) - \int_{1/2}^{\sqrt{3}/2} \frac{y}{\sqrt{1-y^2}}\,dy$$

$$= \left(\frac{\sqrt{3}}{2}\cdot\frac{\pi}{3} - \frac{1}{2}\cdot\frac{\pi}{6}\right) - \int_{1/2}^{\sqrt{3}/2} \frac{y}{\sqrt{1-y^2}}\,dy$$

$$= \left(\frac{\sqrt{3}\pi}{6} - \frac{\pi}{12}\right) + 2\int_{3/4}^{1/4} u^{-1/2}\,du = \left(\frac{\sqrt{3}\pi}{6} - \frac{\pi}{12}\right) + 2\,\frac{u^{1/2}}{1/2}\Bigg|_{3/4}^{1/4} = \boxed{\left(\frac{\sqrt{3}\pi}{6} - \frac{\pi}{12}\right) + 4\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)}.$$

$u = 1-y^2$	$y\,dy =$	$y = \frac{1}{2} \to u = 3/4$
$du = -\frac{1}{2}y\,dy$	$-2\,du$	$y = \frac{\sqrt{3}}{2} \to u = 1/4$