2/10
Mon

8.1

Def: A sequence is an infinite list of numbers, written in a definite order:

$$a_1, a_2, a_3, a_4, \ldots$$

a_n is called the nth term of the sequence.

We also denote sequences like this: $\{a_n\}$ or $\{a_n\}_{n=1}^{\infty}$

Ex:

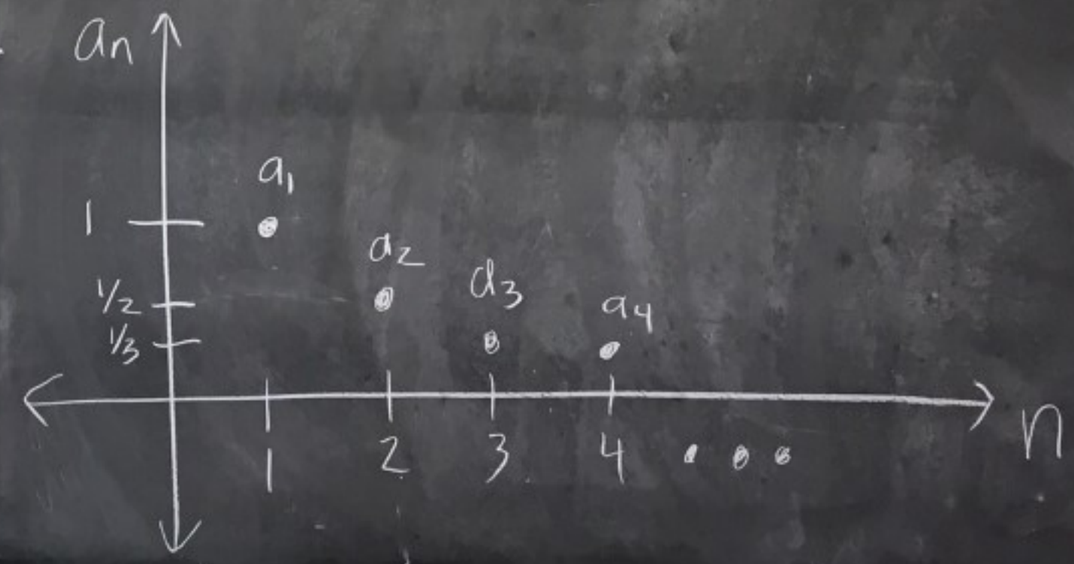
$0, 42, -400, 1, \ldots$

$a_1, a_2, a_3, a_4, \ldots$

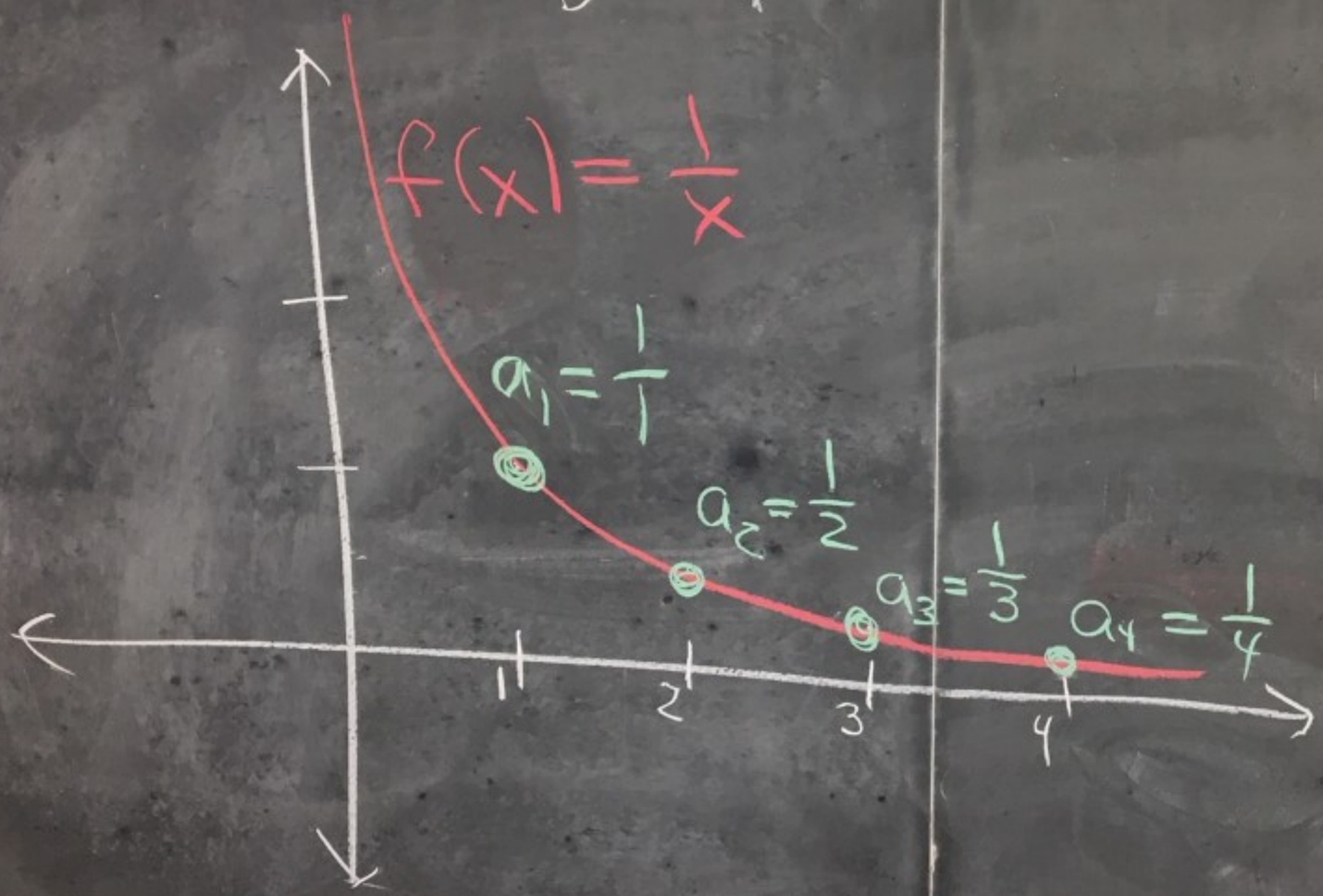
Ex: $a_n = \frac{1}{n}$, $n \geq 1$

$a_1, a_2, a_3, a_4, a_5, \ldots$

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$



This sequence "lives" on the graph of $f(x) = \frac{1}{x}$.



Ex:

$$\left\{(-1)^n \frac{5n^2}{n+1}\right\}_{n=1}^{\infty}$$

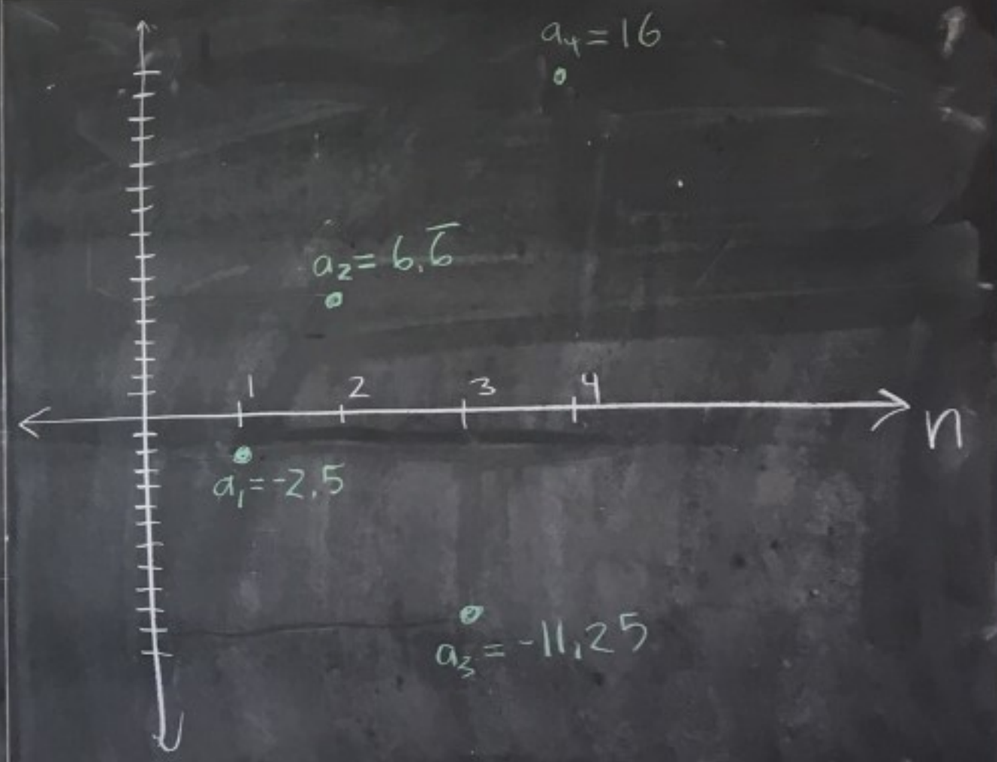
This means $a_n = (-1)^n \frac{5n^2}{n+1}$, $n \geq 1$

$a_1 = (-1)^1 \cdot \frac{5(1)^2}{1+1} = -\frac{5}{2} = -2.5$

$a_2 = (-1)^2 \cdot \frac{5(2)^2}{2+1} = \frac{20}{3} = 6.\overline{6}$

$a_3 = (-1)^3 \cdot \frac{5(3)^2}{3+1} = -\frac{45}{4} = -11.25$

$a_4 = (-1)^4 \cdot \frac{5(4)^2}{4+1} = \frac{80}{5} = 16$



Ex: Find a formula for the following sequence.

$$\frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}, \frac{1}{6}, \dots$$

a_1 a_2 a_3 a_4 a_5, ...

Th

a_1

a_2

$$a_n = (-1)^{n+1} \cdot \frac{1}{n+1}$$

Test:

$$a_1 = (-1)^{1+1} \cdot \frac{1}{1+1} = (-1)^2 \cdot \frac{1}{2} = \frac{1}{2} \quad \checkmark$$

$$a_2 = (-1)^{2+1} \cdot \frac{1}{2+1} = (-1)^3 \frac{1}{3} = -\frac{1}{3} \quad \checkmark$$

Def: If the terms of a sequence $\{a_n\}$ approach a unique number L as n increases — that is if a_n can be close to L by taking n to be sufficiently large —

then

$\lim_{n\to\infty}$

the s

to L

If t

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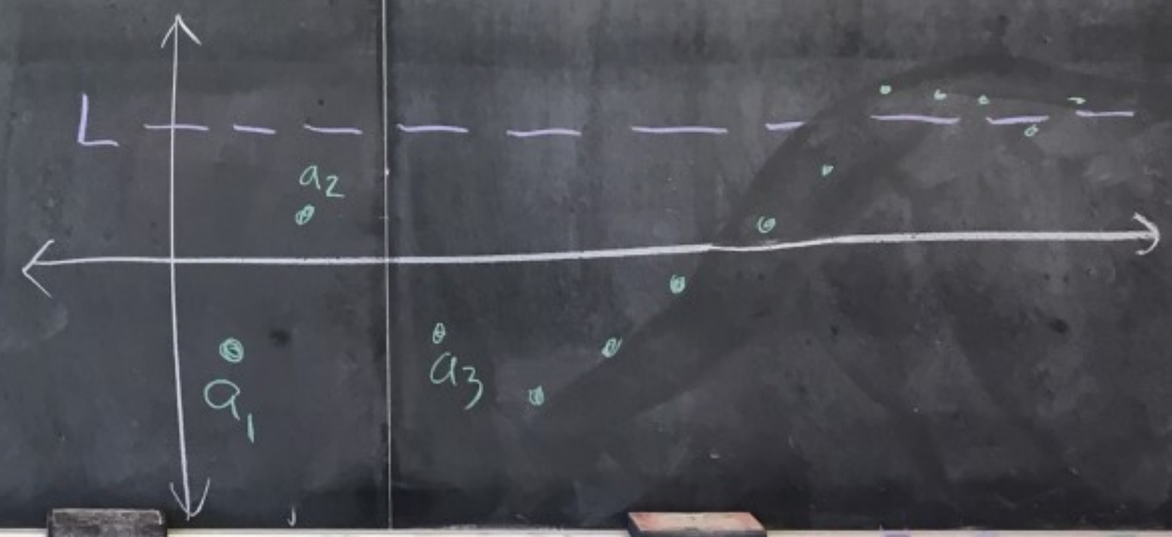
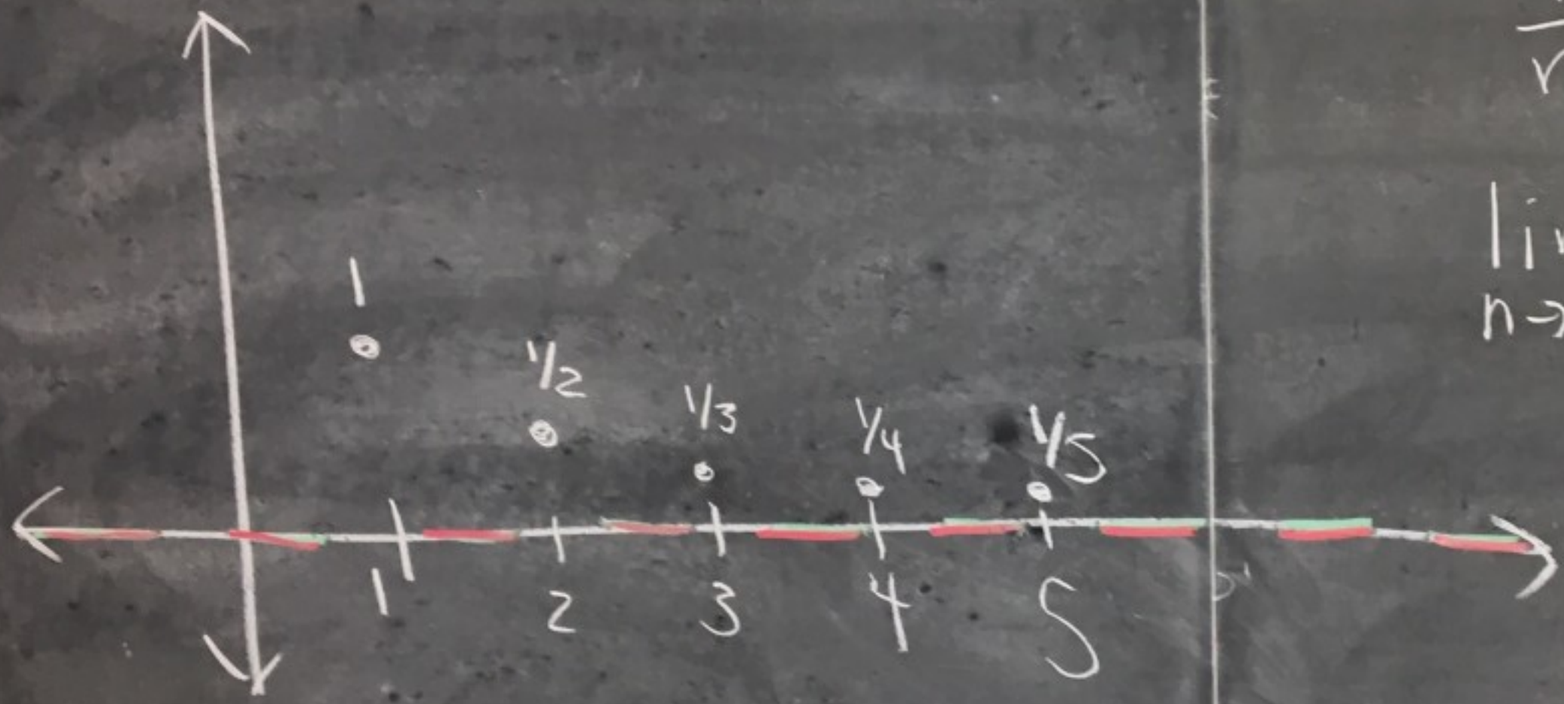
then we say that $\lim_{n\to\infty} a_n = L$, and the sequence converges to L.

If the terms of the sequence do not approach a single number as n increases, then the sequence has no limit and we say it diverges.



Ex: $a_n = \frac{1}{n}$

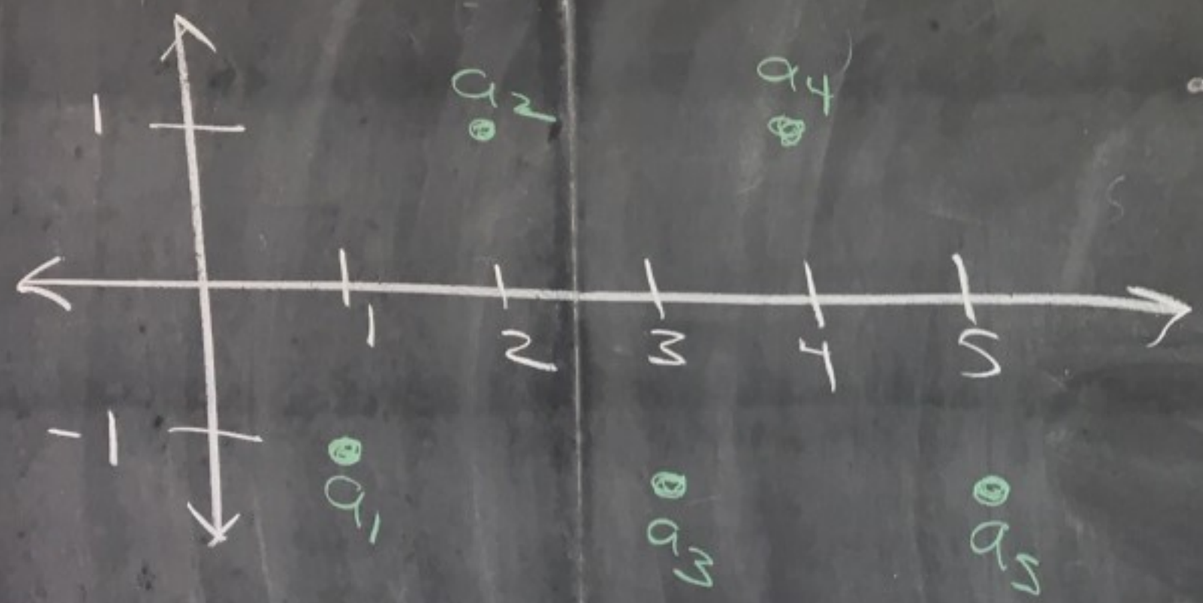


$\frac{1}{n}$ converges

$\lim_{n\to\infty} \frac{1}{n} = 0$

L=0

Ex: $a_n = (-1)^n$



$a_1 = (-1)^1 = -1$

$a_2 = (-1)^2 = 1$

$a_3 = (-1)^3 = -1$

$a_4 = (-1)^4 = 1$

$\vdots$

a_n has no limit as $n \to \infty$.

So, $a_n = (-1)^n$ diverges