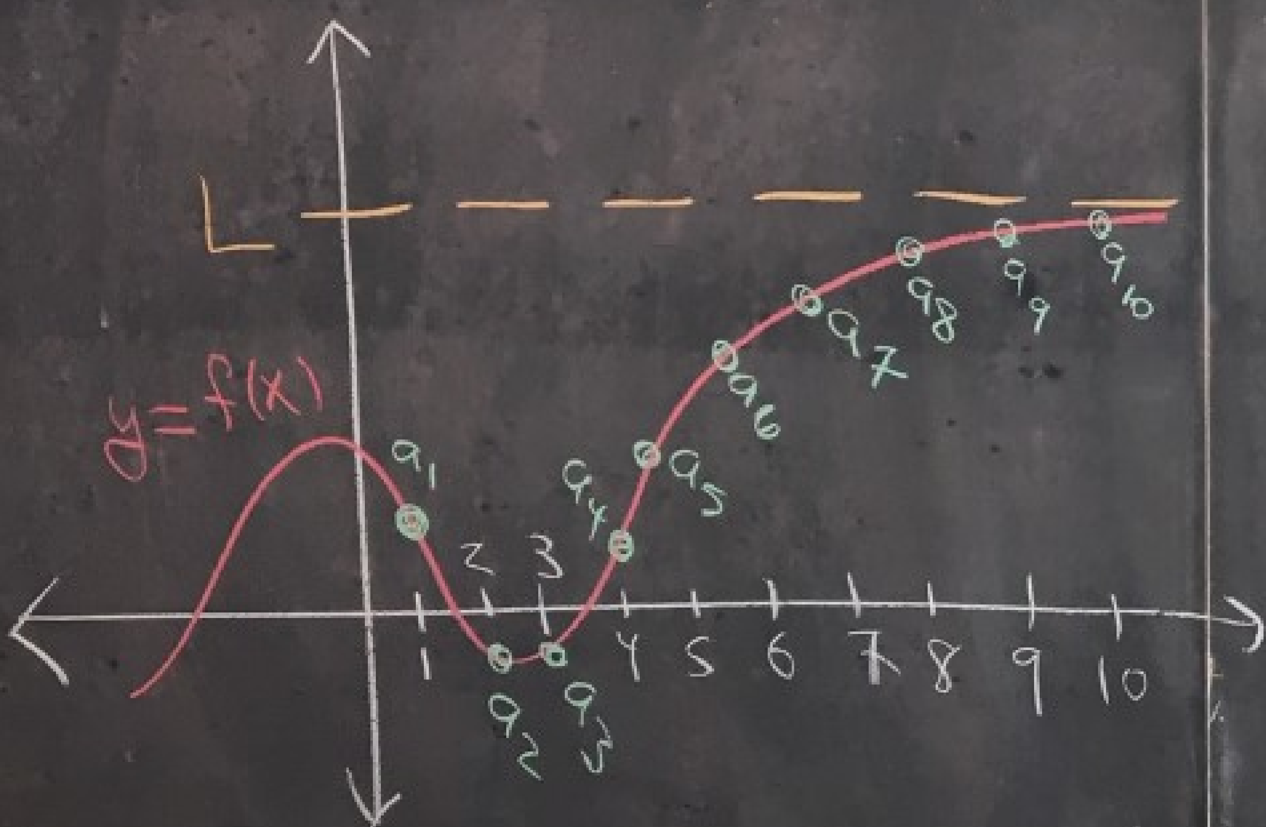
Tuesday
2/11



8.2 - sequences

Thm Suppose $f(x)$ is a function where $f(n) = a_n$ for all $n \geq 1$.

If $\lim_{x\to\infty} f(x) = L$, then $\lim_{n\to\infty} a_n = L$.

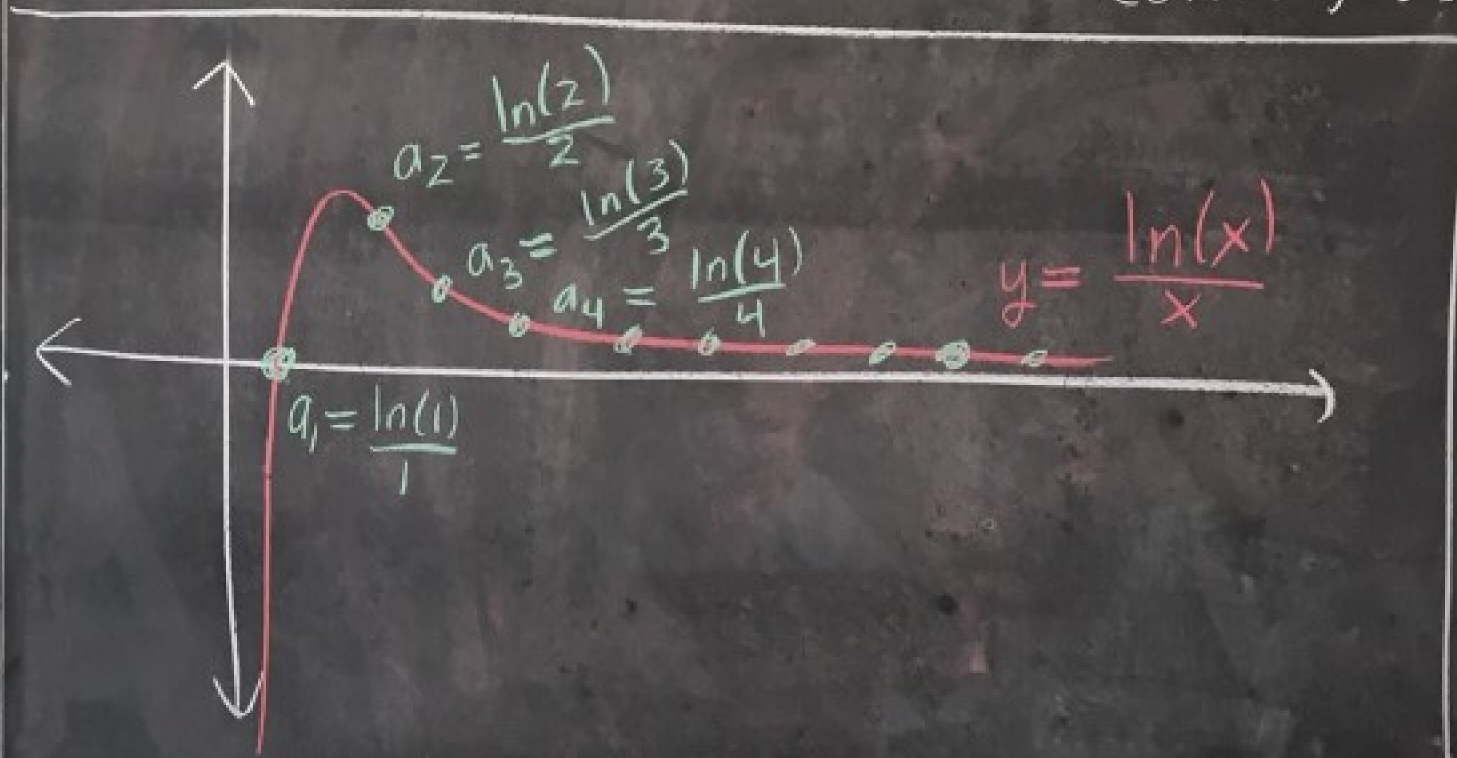
Ex: Does $\left\{ \frac{\ln(n)}{n} \right\}_{n=1}^{\infty}$ converge or diverge?

Let $f(x) = \frac{\ln(x)}{x}$.

Then $f(n) = \frac{\ln(n)}{n}$ for any integer $n \geq 1$.

$$\lim_{x\to\infty} \frac{\ln(x)}{x} \underset{\uparrow\ \text{“}\frac{\infty}{\infty}\text{”}}{\overset{\text{L'H}}{=}} \lim_{x\to\infty} \frac{\left(\frac{1}{x}\right)}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$$

Therefore, $\lim_{n\to\infty} \frac{\ln(n)}{n} = 0$, So, $\left\{ \frac{\ln(n)}{n} \right\}$ converges.



Thm: Suppose that $\{a_n\}$ converges to A and $\{b_n\}$ converges to B.

Then

1. $\lim_{n\to\infty}(a_n+b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n = A+B$
2. $\lim_{n\to\infty}(a_n-b_n) = \lim_{n\to\infty} a_n - \lim_{n\to\infty} b_n = A-B$
3. If c is a constant, then $\lim_{n\to\infty}(c\,a_n) = c\lim_{n\to\infty} a_n = cA$

$< A$

④ $$\lim_{n\to\infty}(a_n b_n) = \left(\lim_{n\to\infty} a_n\right)\left(\lim_{n\to\infty} b_n\right) = AB$$

⑤ If $B \neq 0$, then $$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n} = \frac{A}{B}$$

Ex: $$\lim_{n\to\infty} \frac{1}{n^2} = \lim_{x\to\infty} \frac{1}{x^2} = 0$$

If $p>0$, $$\lim_{n\to\infty} \frac{1}{n^p} = 0$$

Ex: $$\lim_{n\to\infty} \frac{5n^3+n}{-n^3+n^2+1} = \lim_{n\to\infty}\left(\frac{5+\frac{1}{n^2}}{-1+\frac{1}{n}+\frac{1}{n^3}}\right)$$

multiply top/bottom by $1/n^3$

$$= \frac{\lim_{n\to\infty} 5 + \lim_{n\to\infty} \frac{1}{n^2}}{\lim_{n\to\infty}(-1) + \lim_{n\to\infty} \frac{1}{n} + \lim_{n\to\infty} \frac{1}{n^3}} = \frac{5+0}{-1+0+0} = -5$$

Geometric sequences

$\boxed{|r|<1}$

Let r be a real number.

$$\lim_{n\to\infty} r^n = \begin{cases} 0, & \text{if } \boxed{-1<r<1} \\ 1, & \text{if } r=1 \\ \text{does not exist}, & \text{if } r\le -1 \text{ or } 1<r \end{cases}$$

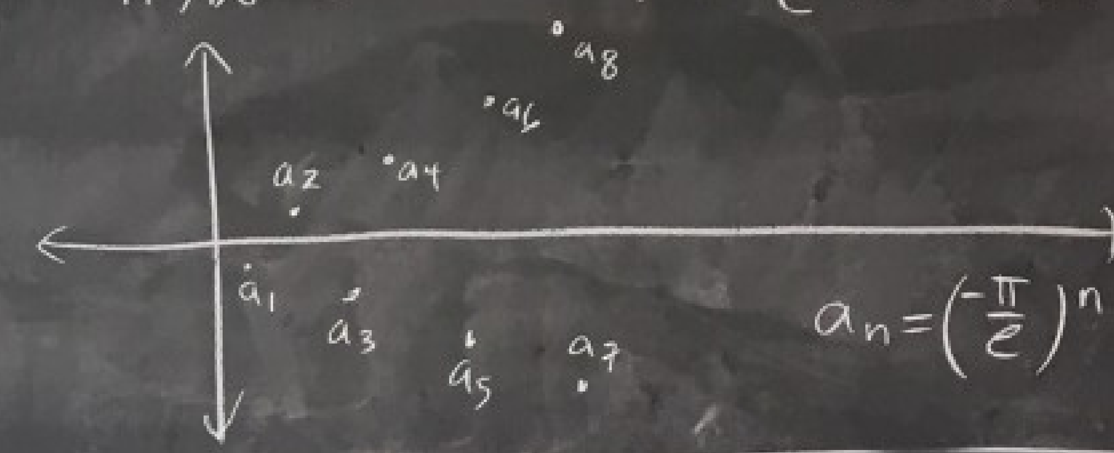
Ex:

$\lim_{n\to\infty} 5^n$ does not exist, $r=5>1$

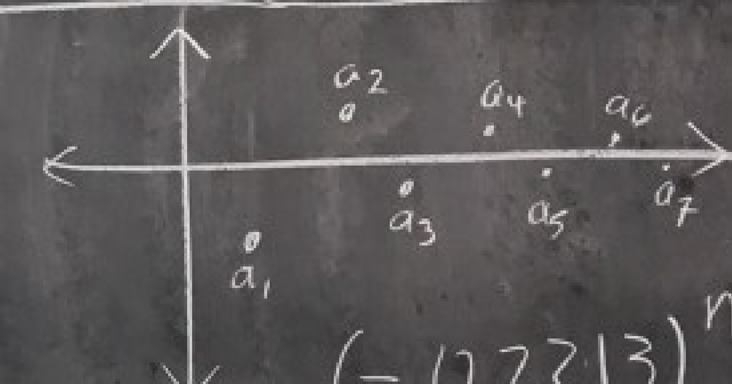
$\lim_{n\to\infty} \left(\frac{3}{7}\right)^n = 0$, $r=\frac{3}{7}$, $-1<r<1$

$\lim_{n\to\infty} (-0.2213)^n = 0$, $r=-0.2213$, $-1<r<1$

$\lim_{n\to\infty} \left(\frac{-\pi}{e}\right)^n$ does not exist
$r = \frac{-\pi}{e} \approx -1.1157$, $r \le -1$



$a_n = \left(\frac{-\pi}{e}\right)^n$



$(-0.2213)^n = a_n$

Squeeze Thm

Let $\{a_n\}$, $\{b_n\}$, and $\{c_n\}$ be sequences.

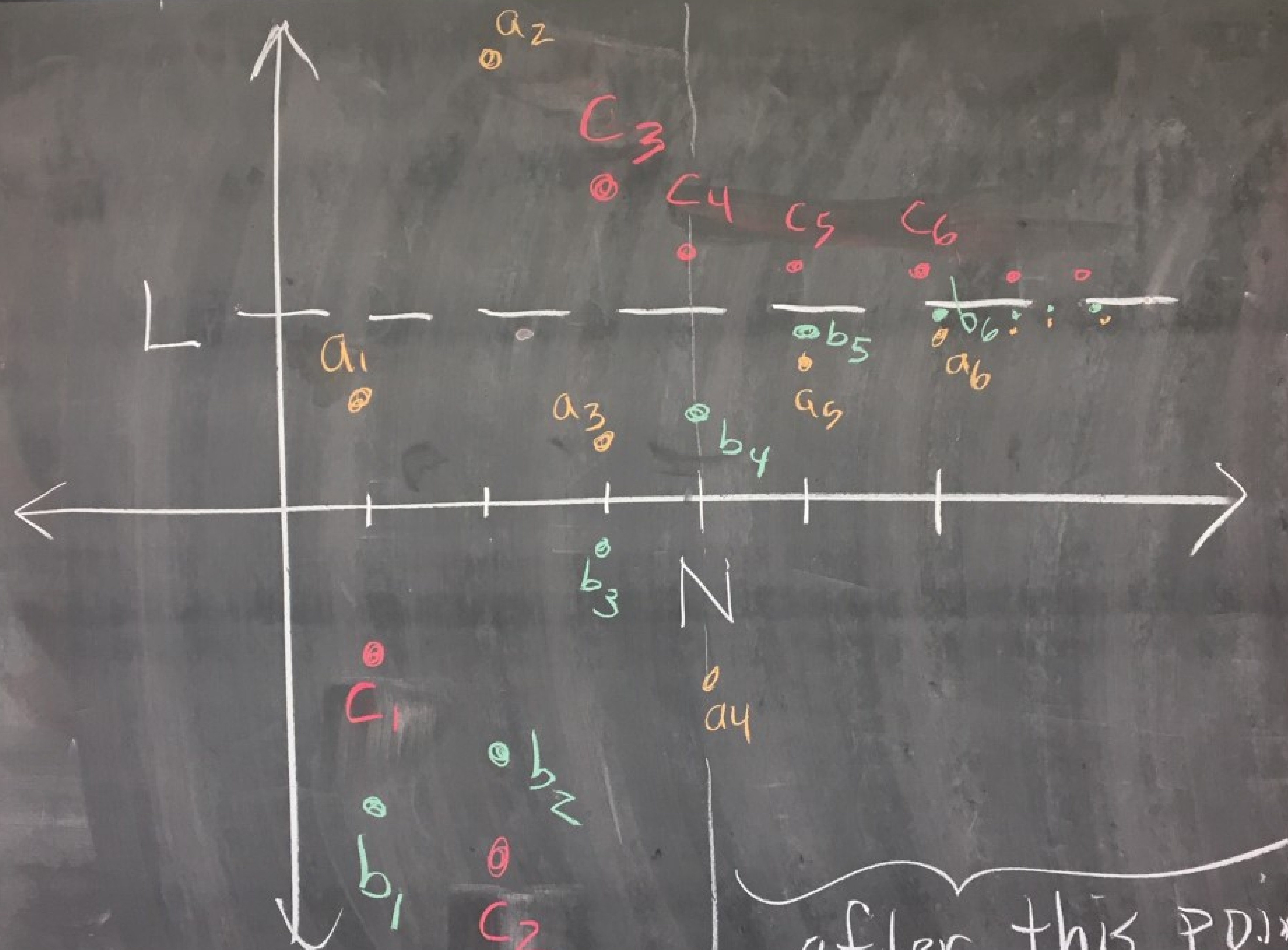
If $a_n \leq b_n \leq c_n$ for all n greater than some point N and $\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} c_n$

then $\lim_{n\to\infty} b_n = L$.

es.

n

-n



after this point

$a_n \leq b_n \leq c_n$