

2/17  
Mon

Test 1 - Weds, 3/4  
covers chapter 7

8.3 continued...

Given  $\sum_{k=1}^{\infty} a_k$  define the partial sums

$$s_1 = a_1$$

$$s_2 = a_1 + a_2$$

$$s_3 = a_1 + a_2 + a_3$$

$\vdots$

$$s_n = a_1 + a_2 + \dots + a_n$$



$$\sum_{k=1}^{\infty} a_k = \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \sum_{k=1}^n a_k$$

Geometric sum formula

If  $r \neq 1$  then

$$1 + r + r^2 + \dots + r^n = \frac{r^{n+1} - 1}{r - 1}$$

⇒ Why is this formula true?

$$(1 + r + r^2 + \dots + r^n)(r - 1)$$

$$= \frac{r + r^2 + r^3 + \dots + r^n + r^{n+1} - 1 - r - r^2 - r^3 - \dots - r^n}{-1 + r^{n+1}}$$

$$\text{So, } (1 + r + r^2 + \dots + r^n)(r - 1) = r^{n+1} - 1$$

Then divide by  $r - 1$ .



Ex: Does

$\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k$  converge  
or diverge?

$$\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k = \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \dots$$

partial sum

$$S_1 = \frac{1}{2}$$

$$S_2 = \frac{1}{2} + \left(\frac{1}{2}\right)^2$$

$$S_3 = \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3$$

$$S_n = \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \dots + \left(\frac{1}{2}\right)^n$$



$$S_n = \frac{1}{2} \left[ 1 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \dots + \left(\frac{1}{2}\right)^{n-1} \right]$$

$$= \frac{1}{2} \left[ \frac{\left(\frac{1}{2}\right)^n - 1}{\frac{1}{2} - 1} \right] = \frac{1}{2} \left[ \frac{\left(\frac{1}{2}\right)^n - 1}{-\frac{1}{2}} \right]$$

$$r = \frac{1}{2}$$

$$= -\left(\frac{1}{2}\right)^n + 1 = 1 - \left(\frac{1}{2}\right)^n$$

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{1}{2}\right)^k = \lim_{n \rightarrow \infty} \left[ 1 - \left(\frac{1}{2}\right)^n \right] = 1 - 0 = 1$$

$$-1 < \frac{1}{2} < 1$$
$$\lim_{n \rightarrow \infty} \left(\frac{1}{2}\right)^n = 0$$

So,  $\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k$   
converges to 1.



## Geometric sums

Consider  $\sum_{k=0}^{\infty} r^k = 1 + r + r^2 + r^3 + \dots$

where  $r$  is a constant.

The partial sums are

$$S_n = 1 + r + r^2 + \dots + r^n$$

Case 1:  $r = 1$

$$\begin{aligned} S_n &= 1 + r + r^2 + \dots + r^n \\ &= \underbrace{1 + 1 + 1 + \dots + 1}_n = n \end{aligned}$$

$$\text{So, } \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} n = \infty$$

Thus,  $\sum_{k=0}^{\infty} 1^k = 1 + 1 + 1 + \dots$  diverges



Case 2:  $r \neq 1$

$$S_n = 1 + r + r^2 + \dots + r^n$$
$$= \frac{r^{n+1} - 1}{r - 1}$$

$$\lim_{n \rightarrow \infty} r^{n+1} = \begin{cases} 0, & \text{if } -1 < r < 1 \\ \text{diverges,} & \text{otherwise} \end{cases}$$

So,

$$\lim_{n \rightarrow \infty} \frac{r^{n+1} - 1}{r - 1} = \begin{cases} \frac{0 - 1}{r - 1} = \frac{1}{1 - r} \\ \text{diverges, otherwise} \end{cases}$$

$\frac{1}{1-r}, -1 < r < 1$   
diverges, otherwise

$$\frac{0 - 1}{r - 1} = \frac{1}{1 - r}$$

Geometric series

$$\sum_{k=0}^{\infty} r^k = 1 + r + r^2 + \dots = \frac{1}{1 - r}$$

if  $-1 < r < 1$ . Otherwise the infinite sum diverges



Ex:  $\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k = 1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \dots$

converge or diverge?

$r = \frac{2}{3}$ ,  $-1 < r < 1$ . So converges.

$$\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k = \frac{1}{1 - \frac{2}{3}} = \boxed{3}$$

If  $-1 < r < 1$ ,  
 $1 + r + r^2 + \dots = \frac{1}{1-r}$



Ex:  $\sum_{k=0}^{\infty} \left(\frac{\pi}{e}\right)^k = 1 + \frac{\pi}{e} + \left(\frac{\pi}{e}\right)^2 + \left(\frac{\pi}{e}\right)^3 + \dots$

$r = \frac{\pi}{e} \approx 1.1557$

$r > 1$

So,  $\sum_{k=0}^{\infty} \left(\frac{\pi}{e}\right)^k$  diverges

Ex:  $\sum_{k=0}^{\infty} \left(-\frac{10}{3}\right)^k = 1 - \frac{10}{3} + \left(\frac{10}{3}\right)^2 - \left(\frac{10}{3}\right)^3 + \dots$

$r = -\frac{10}{3} < -1$   
diverges

Ex:

$= 1$

$S_1 =$

$S_2 =$

$S_3 =$

$S_4 =$

$S_5 =$

lim

$n \rightarrow \infty$



Ex:  $\sum_{k=0}^{\infty} (-1)^k$

$= 1 - 1 + 1 - 1 + 1 - 1 + 1 - \dots$

$S_1 = 1$

$S_2 = 1 - 1 = 0$

$S_3 = 1 - 1 + 1 = 1$

$S_4 = 1 - 1 + 1 - 1 = 0$

$S_5 = 1 - 1 + 1 - 1 + 1 = 1$

$\lim_{n \rightarrow \infty} S_n$  does not exist

$\sum_{k=0}^{\infty} (-1)^k$  diverges

Emergency  
CAL STATE L.A.  
CALIFORNIA STATE UNIVERSITY, LOS ANGELES

**EVACUATION**  
Evacuate the building using the nearest exit or elevator if ordered or instructed.  
Do not use elevators, stairs or escalators.  
Do not re-enter the building until authorized.  
Do not use the building as a shelter.  
Do not use the building as a storage area.  
Do not use the building as a meeting place.  
Do not use the building as a classroom.  
Do not use the building as a laboratory.  
Do not use the building as a workshop.  
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Do not use the building as a laboratory.  
Do not use the building as a workshop.

**911**  
PUBLIC SAFETY  
POLICE FIRE AMBULANCE  
ENVIRONMENTAL HEALTH AND SAFETY  
FACILITY SERVICES

**SHELTER IN PLACE**  
Stay inside the building as ordered by a safe shelter.  
If you are in a room with a door, close the door to block the entry of contaminants. Stay in the room until instructed to leave.  
If you are in a room with a window, cover the window with a wet cloth.  
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**POWER OUTAGE**  
Remain calm.  
Do not touch electrical equipment.  
Do not use electrical equipment.  
Do not use electrical equipment.  
Do not use electrical equipment.  
Do not use electrical equipment.  
Do not use electrical equipment.  
Do not use electrical equipment.

**HAZARDOUS MATERIALS**  
Do not touch or taste any substance.  
Do not breathe any dust or fumes.  
Do not get any substance on your skin or clothes.  
Do not get any substance in your eyes or mouth.  
Do not get any substance on your face.  
Do not get any substance on your hair.  
Do not get any substance on your hands.  
Do not get any substance on your feet.

**FIRE**  
Do not panic.  
Do not run.  
Do not use the elevator.  
Do not use the stairs.  
Do not use the stairs.  
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