2/18
Tuesday

8.3 continued...

Last time

Geometric series

$$\sum_{k=0}^{\infty} r^k = 1 + r + r^2 + r^3 + \dots \underset{\boxed{-1<r<1}}{=} \frac{1}{1-r}$$

converges: $-1<r<1$
diverges: otherwise

8.3

(28) Converge or diverge?

$$\sum_{k=3}^{\infty} \frac{3\cdot 4^k}{7^k} = 3\cdot\frac{4^3}{7^3} + 3\cdot\frac{4^4}{7^4} + 3\cdot\frac{4^5}{7^5} + \cdots$$

$$= 3\left[\left(\frac{4}{7}\right)^3 + \left(\frac{4}{7}\right)^4 + \left(\frac{4}{7}\right)^5 + \cdots\right]$$

$$= 3\cdot\left(\frac{4}{7}\right)^3\left[1 + \left(\frac{4}{7}\right)^1 + \left(\frac{4}{7}\right)^2 + \left(\frac{4}{7}\right)^3 + \cdots\right]$$

converges

$r = \frac{4}{7}$

$-1 < r < 1$

$$= 3\cdot\left(\frac{4}{7}\right)^3\left[\frac{1}{1-\frac{4}{7}}\right]$$

$$= 3\left(\frac{4}{7}\right)^3\cdot\frac{7}{3} = \frac{4^3}{7^2} = \boxed{\frac{64}{49}}$$

$\boxed{8.4}$ – Divergence and Integral tests

Divergence Test

If $\lim_{k\to\infty} a_k \neq 0$,

then $\sum_{k=1}^{\infty} a_k$ diverges

Ex: Does

$$\sum_{k=0}^{\infty} \frac{k}{2k+1} =$$

$$\lim_{k\to\infty} \frac{k}{2k+1} =$$

al tests

Ex: Does $\displaystyle\sum_{k=0}^{\infty} \frac{k}{2k+1}$ converge or diverge?

$$\sum_{k=0}^{\infty} \frac{k}{2k+1} = 0 + \frac{1}{3} + \frac{2}{5} + \frac{3}{7} + \frac{4}{9} + \frac{5}{11} + \frac{7}{13} + \cdots$$

$$\lim_{k\to\infty} \frac{k}{2k+1} = \lim_{k\to\infty} \frac{1}{2+\frac{1}{k}} = \frac{1}{2+0} = \frac{1}{2} \neq 0$$

multiply by $\frac{1}{k}$ top/bottom

Since $\displaystyle\lim_{k\to\infty} \frac{k}{2k+1} \neq 0$

by the divergence test

$\displaystyle\sum_{k=0}^{\infty} \frac{k}{2k+1}$ diverges.

egral tests

$$\boxed{\lim_{k\to\infty} r^k = 0 \text{ if } -1<r<1}$$

Note: If $\lim_{k\to\infty} a_k = 0$

the divergence test doesn't apply.

You need another test to determine whether or not $\sum_{k=1}^{\infty} a_k$ converges or diverges.

E

$k=$

We

co

Ho

Her

Ex: Consider

$$\sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^k = \frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\dots$$

We saw that this series converges to 1.

However

$$\lim_{k\to\infty}\left(\frac{1}{2}\right)^k = 0 \qquad a_k=\left(\frac{1}{2}\right)^k$$

es.

Here $\lim\limits_{k\to\infty} a_k = 0$ and $\sum\limits_{k=1}^{\infty} a_k$ converges.

Ex: Harmonic series

$$\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$$

Converge or diverge?

Note: $\lim_{k\to\infty} \frac{1}{k} = 0$ however $\sum_{k=1}^{\infty} \frac{1}{k}$ diverges

Nic

$\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2}$

$\geq 1 +$

$= 1 +$

So, $\sum_{k}^{\infty}$

Nicole Oresme (1300s - Middle Ages)

$$\sum_{k=1}^{\infty}\frac{1}{k} = 1+\frac{1}{2}+\underbrace{\frac{1}{3}+\frac{1}{4}}+\underbrace{\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}}+\underbrace{\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+\frac{1}{12}+\frac{1}{13}+\frac{1}{14}+\frac{1}{15}+\frac{1}{16}}+\cdots$$

$$\geqslant 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\cdots$$

$$= 1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\cdots$$

diverges

So, $\sum_{k=1}^{\infty}\frac{1}{k}$ must diverge since its "bigger" than $1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\cdots$ which is infinite

Other method

$\sum_{k=1}^{\infty} \frac{1}{k}$ can be thought of as adding up the blue boxes.

Orange is $\int_1^{\infty} \frac{1}{x}\,dx$

So, if $\sum \frac{1}{k}$ converged we'd have:

$$\sum_{k=1}^{\infty} \frac{1}{k} > \int_1^{\infty} \frac{1}{x}\,dx = \lim_{t\to\infty} \int_1^{t} \frac{1}{x}\,dx$$

$$= \lim_{t\to\infty} \ln|x| \Big|_1^{t}$$

$$= \lim_{t\to\infty} \underbrace{\ln|t|}_{\infty} - \underbrace{\ln|1|}_{0} = \infty$$

So, $\sum_{k=1}^{\infty} \frac{1}{k}$ must diverge to ∞