2/26
Weds
week 6

Ex: Does $\sum_{n=1}^{\infty} \frac{10}{2n^3+n}$

converge or diverge?

Does the divergence test apply?

$$\lim_{n\to\infty} \frac{10}{2n^3+n} = 0$$

So divergence test does not apply.

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$\frac{10}{2n^3}$

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ply?

apply.

Spider sense

When n is large,

$$\frac{10}{2n^3+n} \approx \frac{10}{2n^3} = \frac{5}{n^3}$$

We know $\sum \frac{1}{n^3}$ converges ($p=3>1$ series)

So we guess $\sum \frac{10}{2n^3+n}$ converges

Note

$$\frac{10}{2n^3+n} < \frac{10}{2n^3} = \frac{5}{n^3}$$

$$\boxed{2n^3+n > 2n^3}$$

Since $\sum_{n=1}^{\infty} \frac{5}{n^3} = 5\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges ($p=3>1$ series)

and $\frac{10}{2n^3+n} < \frac{5}{n^3}$ by the comparison test $\sum_{n=1}^{\infty} \frac{10}{2n^3+n}$ converges.

Ex: Does $\sum_{n=2}^{\infty} \frac{\ln(n)}{n} = \frac{\ln(2)}{2} + \frac{\ln(3)}{3} + \cdots$

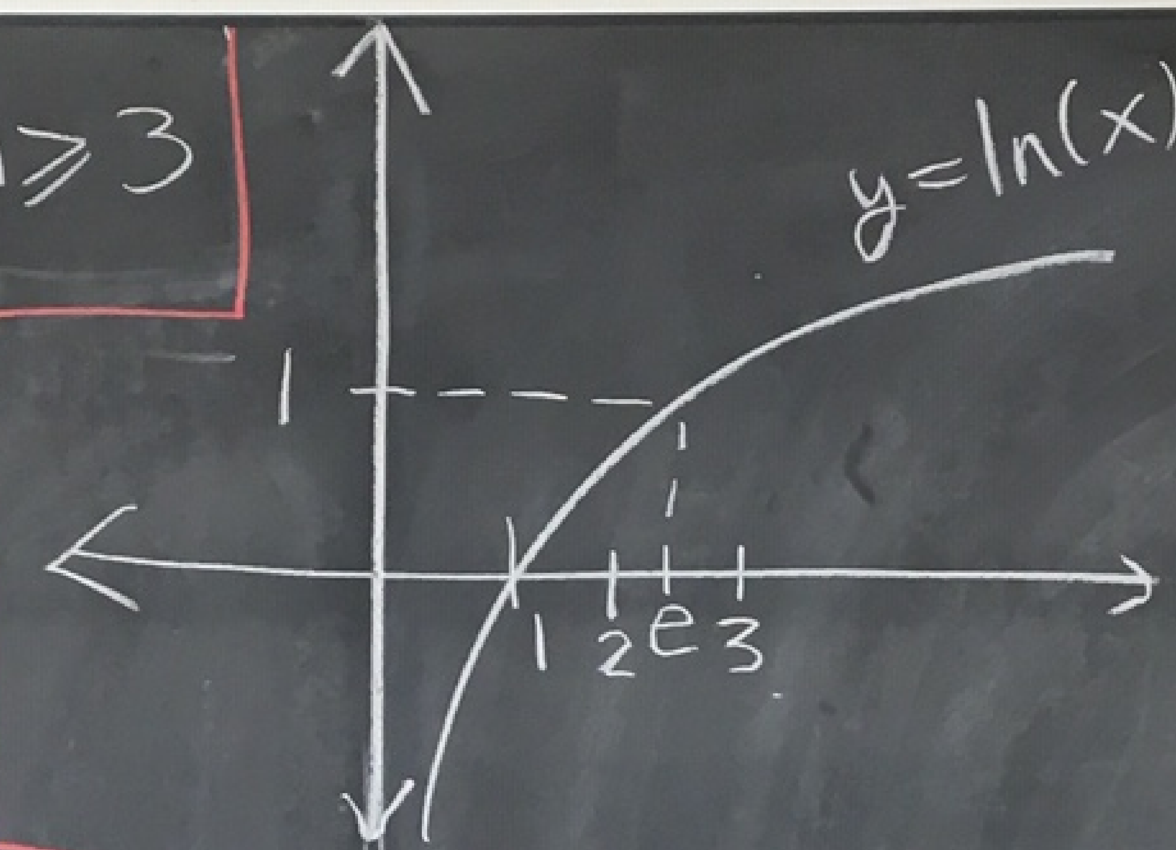
converge or diverge?

Does the divergence test apply? $\boxed{\text{NO}}$

$$\lim_{n\to\infty} \frac{\ln(n)}{n} \underset{\text{"}\frac{\infty}{\infty}\text{"}}{\overset{L'H}{=}} \lim_{n\to\infty} \frac{1/n}{1} = \lim_{n\to\infty} \frac{1}{n} = 0$$

Note that $\ln(n) > 1$ when $n \geqslant 3$

So, $\frac{\ln(n)}{n} > \frac{1}{n}$ when $n \geqslant 3$..



By the comparison test

$\sum_{n=3}^{\infty} \frac{\ln(n)}{n}$ diverges since $\sum_{n=3}^{\infty} \frac{1}{n}$ diverges.

So, $\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$ diverges.

Note * * *

When testing if a series converges or diverges, it doesn't matter where you start the series.

That is, $\sum_{n=b}^{\infty} a_n$ and $\sum_{n=c}^{\infty} a_n$

either both converge, or both diverge, no matter what starting points b or c you pick.

Ex: Does $\sum_{k=2}^{\infty} \frac{\ln(k)}{k^3}$ converge or diverge?

Divergence test

$$\lim_{k\to\infty} \frac{\ln(k)}{k^3} \overset{L'H}{=} \lim_{k\to\infty} \frac{1/k}{3k^2} = \lim_{k\to\infty} \frac{1}{3k^3} = 0$$

"$\frac{\infty}{\infty}$"

So, divergence test doesn't apply.

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$\boxed{\text{Claim:}}$ $\boxed{\ln(k) < k \text{ when } k \geq 1.}$

Let $f(x) = x - \ln(x)$.

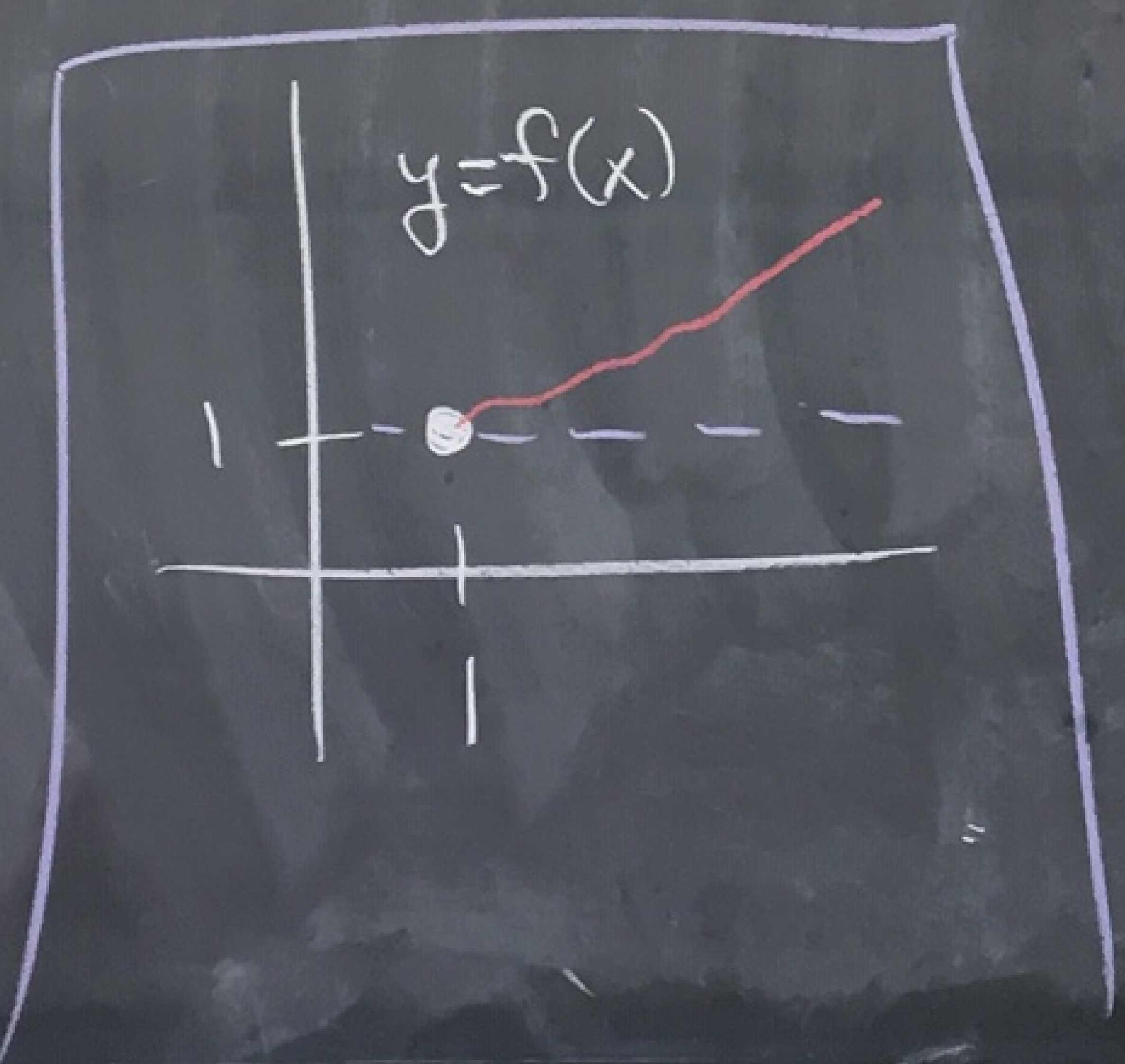
Then, $f(1) = 1 - \ln(1) = 1 - 0 = 1$.

Also,

$$f'(x) = 1 - \frac{1}{x} \geq 0$$

when $x \geq 1$.

So, f never drops below height 1 when $x \geq 1$.



So,

$f(x) = x - \ln(x) \geq 1$ when $x \geq 1$.

So, $x - \ln(x) > 0$ when $x \geq 1$.

Thus, $\boxed{x > \ln(x) \text{ when } x \geq 1.}$

So, $\frac{\ln(k)}{k^3} < \frac{k}{k^3} = \frac{1}{k^2}$ when $k \geq 2$.

And $\sum_{k=2}^{\infty} \frac{1}{k^2}$ is a convergent

$p = 2 > 1$ series.

By the comparison test $\sum_{k=2}^{\infty} \frac{\ln(k)}{k^3}$

converges.