3/10
Tuesday
week 8

Week	M	W
9		
10		3/25 - Test 2
Spring break		
11		
12		
13		4/22 Test 3
14		
15		
FINALS		TUES. 5/12 12-2

Test 2
covers
Chapter
8

R

9.2 continued

Ex: Find the interval and radius of convergence of

$$\sum_{k=1}^{\infty} \frac{(x-2)^k}{\sqrt{k}} = (x-2) + \frac{1}{\sqrt{2}}(x-2)^2 + \frac{1}{\sqrt{3}}(x-3)^3 + \ldots$$

$\boxed{\text{Ratio Test}}$

$$L = \lim_{k\to\infty} \left| \frac{\frac{(x-2)^{k+1}}{\sqrt{k+1}}}{\frac{(x-2)^k}{\sqrt{k}}} \right| = \lim_{k\to\infty} \left| \frac{\sqrt{k}}{(x-2)^k} \cdot \frac{(x-2)^{k+1}}{\sqrt{k+1}} \right|$$

$$= \lim_{k\to\infty} \left| (x-2)\sqrt{\frac{k}{k+1}} \right|$$

$$= \lim_{k\to\infty} |x-2| \left| \sqrt{\frac{1}{1+\frac{1}{k}}} \right| = |x-2|\sqrt{\frac{1}{1+0}} = |x-2|$$

divide top/bottom by k

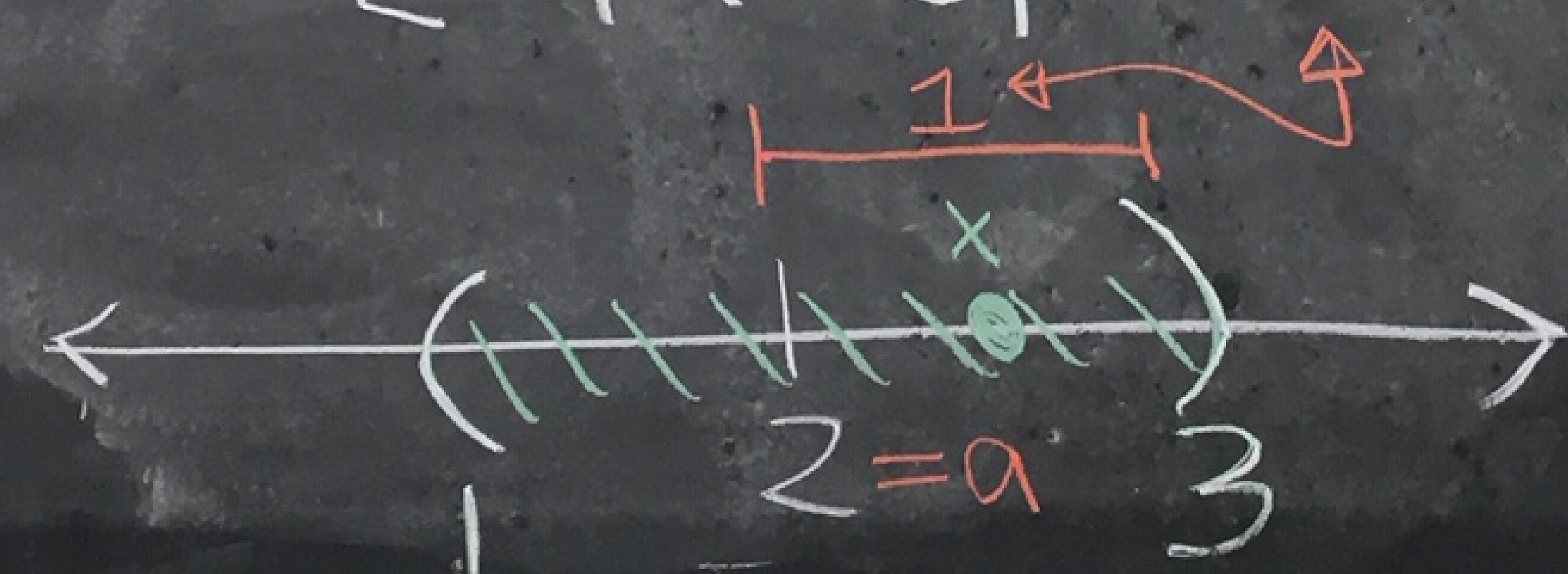
We get absolute convergence if $L = |x-2| < 1$

$|x-a| < c$

$a-c < x < a+c$

$2-1 < x < 2+1$

$1 < x < 3$



Test endpoints

$\boxed{x=1}$: $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}(x-2)^k \underset{\boxed{x=1}}{=} \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}(1-2)^k = \sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$

Alt. series test

$\sum (-1)^k a_k$

① $0 < a_{k+1} \leq a_k$ for all large enough k ← $\boxed{\text{for all } k \geq N \text{ for some } N}$

② $\lim_{k\to\infty} a_k = 0$

$a_k = \frac{1}{\sqrt{k}}$

① $\frac{1}{\sqrt{k+1}} \leq \frac{1}{\sqrt{k}}$ ✓

② $\lim_{k\to\infty} \frac{1}{\sqrt{k}} = 0$ ✓

So, $\sum_{k=1}^{\infty} (-1)^k \frac{1}{\sqrt{k}}$ converges

$\boxed{x=3}$: $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}(x-2)^k \underset{\boxed{x=3}}{=} \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}(3-2)^k$

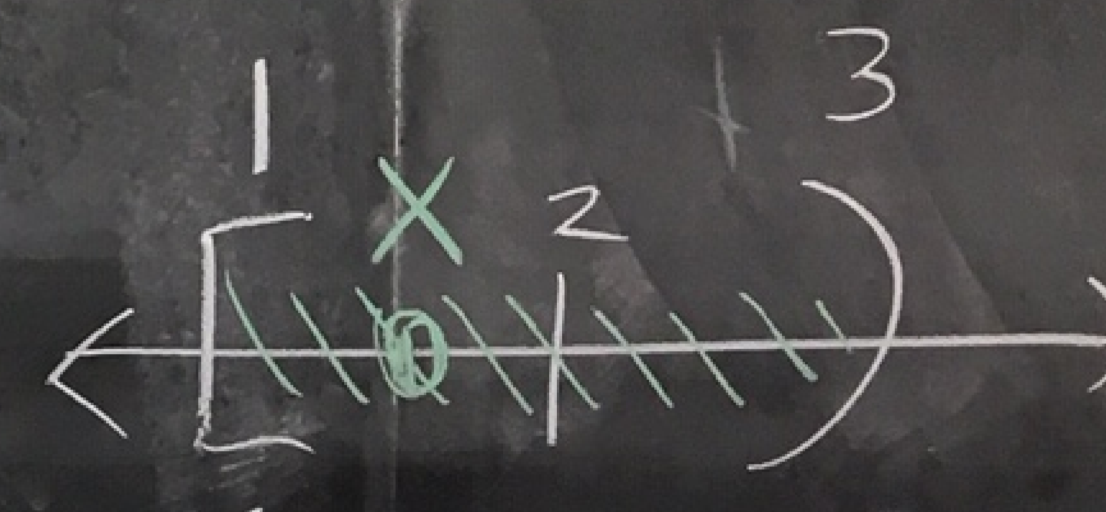
$$= \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}} = \sum_{k=1}^{\infty} \frac{1}{k^{1/2}}$$

$p = 1/2$ series
$p \not> 1$
So this p-series diverges

$\boxed{\text{Answer}}$:

radius of convergence: $R = 1$

interval of convergence: $[1, 3)$ or $1 \le x < 3$ or



Ex: Same question for $\displaystyle\sum_{k=0}^{\infty} k!\,x^k = \underbrace{1}_{\substack{k=0\\0!=1}} + \underbrace{x}_{k=1} + \underbrace{(2!)x^2}_{k=2} + \underbrace{(3!)x^3}_{k=3} + \underbrace{\cdots}_{k>3}$

$\boxed{\text{Ratio test}}$

$$L = \lim_{k\to\infty} \left| \frac{(k+1)!\, x^{k+1}}{k!\, x^k} \right| = \lim_{k\to\infty} \left| \frac{(k+1)(k!)\, x^k\, x}{k!\, x^k} \right|$$

$(k+1)! = (k+1)\cdot(k!)$

$$= \lim_{k\to\infty} |k+1|\cdot|x|$$

Case 1: If $x=0$, then

$$L=\lim_{k\to\infty}|k+1|\cdot|x| = \lim_{k\to\infty} 0 = 0$$

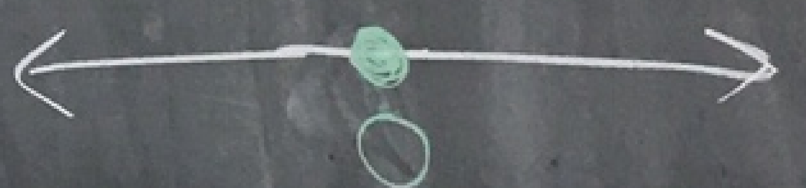
So when $x=0$, $L<1$ and we get convergence.

Case 2: If $x\neq 0$, then

$$L=\lim_{k\to\infty}|k+1|\cdot|x| = \infty$$ and we get divergence.

Answer

$\sum_{k=0}^{\infty}(k!)\cdot x^k$ converges only at $x=0$.



radius of convergence: $R=0$

interval of convergence: $\{0\}$

means the set with just 0

Ex: Express $\frac{1}{1-2x}$ as a power series and find its radius of convergence.

We know (Geometric series)

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots = \sum_{k=0}^{\infty} x^k$$

converges for $\underbrace{|x| < 1}_{-1 < x < 1}$

So,

$$\frac{1}{1-2x} = 1 + (2x) + (2x)^2 + (2x)^3 + \cdots$$

$$= 1 + 2x + 2^2x^2 + 2^3x^3 + \cdots$$

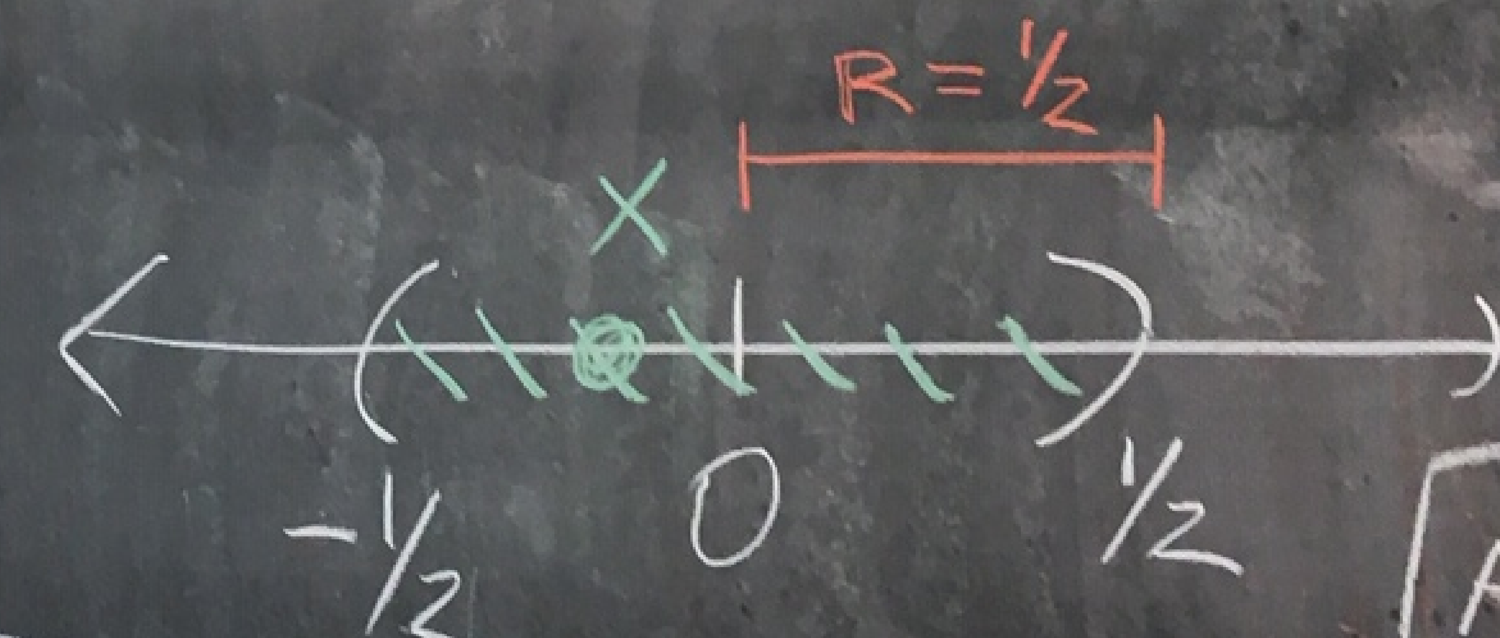
$|2x| < 1$

$2|x| < 1$

$|x| < \frac{1}{2}$

$-\frac{1}{2} < x < \frac{1}{2}$

$R = \frac{1}{2}$



Another way to write it

$$\frac{1}{1-2x} = \sum_{k=0}^{\infty} (2x)^k = \sum_{k=0}^{\infty} 2^k x^k$$