

3/11
Weds
Week 8

9.2
Continued

Ex: Find a power series representation and radius of convergence of

$$f(x) = \frac{x^3}{x+2}$$

$$\frac{1}{1-r} = 1 + r + r^2 + \dots$$

$|r| < 1$

power series
and radius
of

$$f(x) = \frac{x^3}{x+2} = x^3 \left[\frac{1}{2+x} \right] = x^3 \cdot \frac{1}{2} \left[\frac{1}{1 + \frac{x}{2}} \right]$$

$$= \frac{x^3}{2} \left[\frac{1}{1 - \left(-\frac{x}{2}\right)} \right] = \frac{x^3}{2} \left[1 + \left(-\frac{x}{2}\right) + \left(-\frac{x}{2}\right)^2 + \left(-\frac{x}{2}\right)^3 + \dots \right]$$

$$\frac{1}{1-r} = 1 + r + r^2 + \dots$$

$$|r| < 1$$

$$\left| -\frac{x}{2} \right| < 1$$

$$\frac{|x|}{2} < 1$$

$$\begin{aligned} |x| &< 2 \\ -2 &< x < 2 \end{aligned}$$

$$= \frac{x^3}{z} \left[1 - \frac{x}{z} + \frac{x^2}{z^2} - \frac{x^3}{z^3} + \frac{x^4}{z^4} - \dots \right]$$

$$= \frac{x^3}{z} - \frac{x^4}{z^2} + \frac{x^5}{z^3} - \frac{x^6}{z^4} + \frac{x^7}{z^5} - \dots$$

radius of convergence $R=2$

$$-2 < x < 2$$

Other way:

$$f(x) = \frac{x^3}{2} \left[\frac{1}{1 - \left(-\frac{x}{2}\right)} \right]$$

as before
 $|x| < 2$

$$\frac{1}{1-r} = \sum_{k=0}^{\infty} r^k \quad |r| < 1$$

$$\frac{x^3}{2} \sum_{k=0}^{\infty} \left(-\frac{x}{2}\right)^k$$

$$= \frac{x^3}{2} \sum_{k=0}^{\infty} \frac{(-1)^k}{2^k} x^k$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{2^{k+1}} x^{k+3}$$

$$-2 < x < 2$$

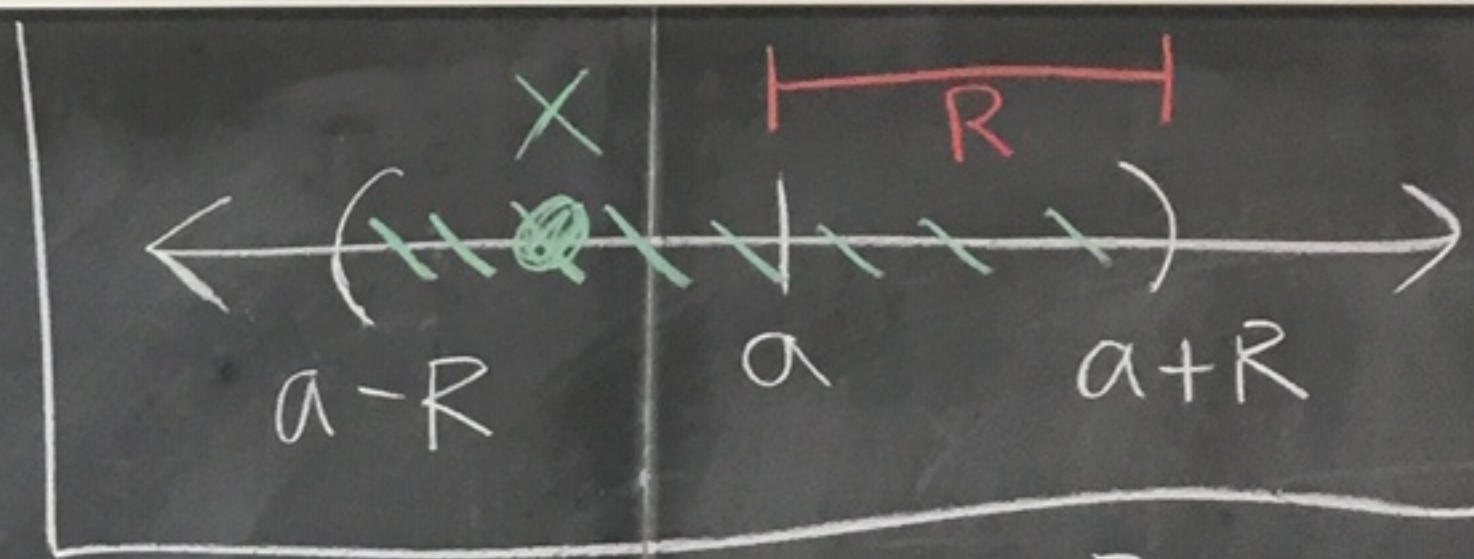
$$(ab)^k = a^k b^k \quad \left(\frac{a}{b}\right)^k = \frac{a^k}{b^k}$$

Theorem: Let $f(x)$ be represented by a power series,

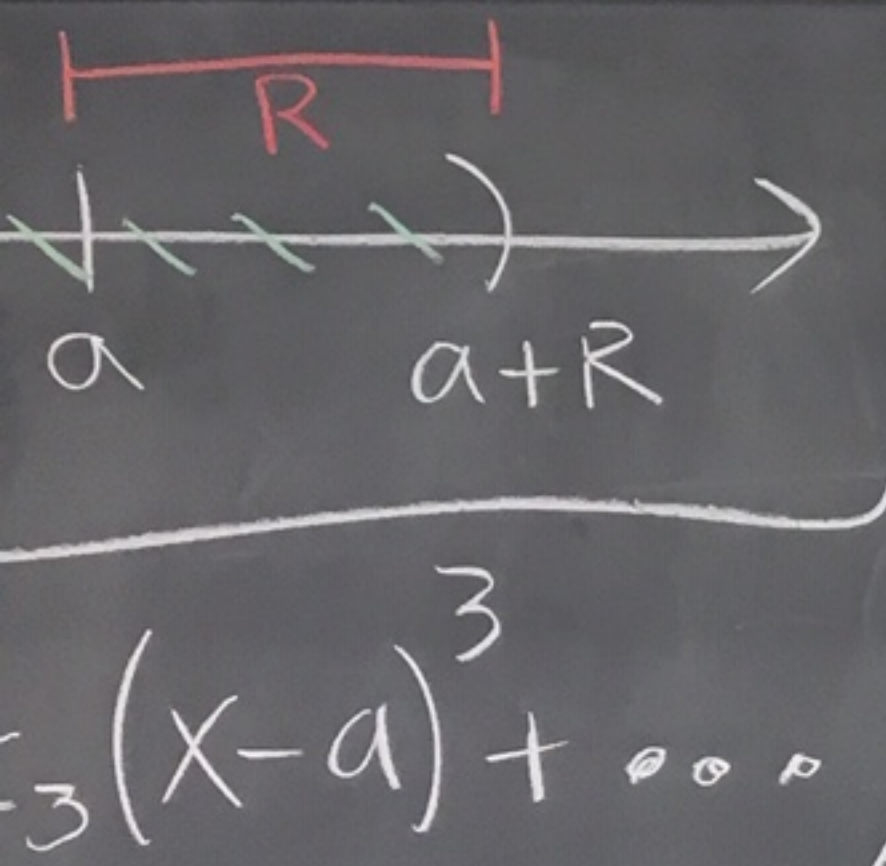
$$f(x) = \sum_{k=0}^{\infty} c_k (x-a)^k = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \dots$$

with radius of convergence R .

Then f is differentiable and continuous on $(a-R, a+R)$ and



*
* Moreover, the
in ① & ② have
of convergence



→ ① $f'(x) =$

$$c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + \dots$$

$$= \sum_{k=1}^{\infty} k \cdot c_k (x-a)^{k-1}$$

* ② $\int f(x) dx =$

$$C + c_0(x-a) + \frac{c_1}{2}(x-a)^2 + \frac{c_2}{3}(x-a)^3 + \dots$$

$$= C + \sum_{k=0}^{\infty} c_k \frac{(x-a)^{k+1}}{k+1}$$

* Moreover, the series in ① & ② have radius of convergence R *

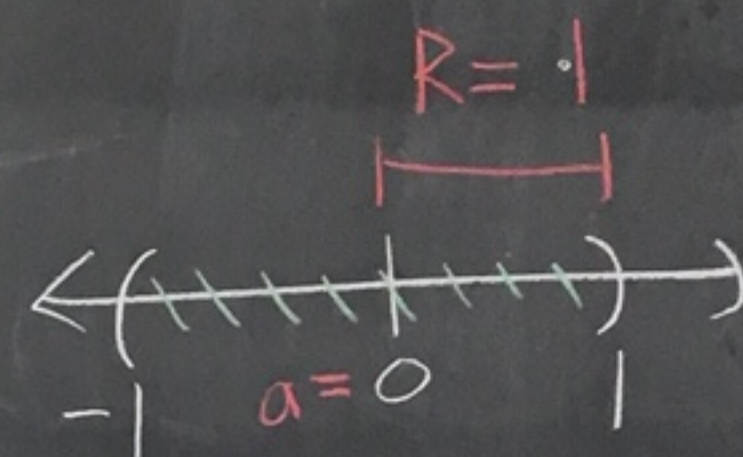


Ex: Notice that $\int \frac{dx}{1-x} = -\ln|1-x| + C$

We know

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots = \sum_{k=0}^{\infty} x^k$$

when $-1 < x < 1$



Integrating we get

$$-\ln(1-x) = C + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5} + \dots = C + \sum_{k=0}^{\infty} \frac{x^{k+1}}{k+1}$$

this will still converge when $-1 < x < 1$

when $-1 < x < 1$
 $0 < 1-x$
So we don't need abs. values in log

So,

$$\ln(1-x) = -C - \sum_{k=0}^{\infty} \frac{x^{k+1}}{k+1}$$

$$= -C - \sum_{k=1}^{\infty} \frac{x^k}{k}$$

Find C, Plug in $x=0$ to get

$$0 = \ln(1) = \ln(1-0) = -C - \sum_{k=1}^{\infty} \frac{0^k}{k} = -C$$

So, $C=0$.

Emergency safety posters on the wall to the right of the chalkboard. The visible posters include:

- EVACUATION**: Instructions for exiting the building.
- SHELTER IN PLACE**: Instructions for staying in the building during an emergency.
- POWER OUTAGE**: Instructions for what to do when power is lost.
- FIRE**: Instructions for fire safety and evacuation.

Thus,

$$\ln(1-x) = - \sum_{k=1}^{\infty} \frac{x^k}{k}$$

when $-1 < x < 1$.