Math

2120-09

3/16/20

Monday

Class + Workshop

1. Canvas webpage
2. Testing

   Move test 2 to 4/8
   Move test 3 to 4/29

3. Test 2 will be taken at home
4. Methods to get the test back to me:

   <u>Best:</u> phone scanner - turns test into pdf.

   - tinyscanner ← free and good
   - camscanner ←

5. HW for before spring break.

Scan some two-three pages (from your calc notes or anything) and email it to me as one pdf document.

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6. These notes will be saved as .pdf and put on the class website.

9.2 continued...

Ex: Find a power series for $\tan^{-1}(x)$ centered at $a=0$ and find it's radius of convergence.

$|-x^2|<1$
$|x|^2<1$
$|x|<1$

Recall:

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{k=0}^{\infty} (-x^2)^k$$

$$= \sum_{k=0}^{\infty} (-1)^k x^{2k}$$

$$= \underset{k=0}{1} - \underset{k=1}{x^2} + \underset{k=2}{x^4} - \underset{k=3}{x^6} + \dots$$

$$\boxed{\left.\begin{aligned} \frac{1}{1-r} &= \sum_{k=0}^{\infty} r^k \\ &= 1 + r + r^2 + r^3 + \dots \end{aligned}\right\} |r|<1}$$

$$\frac{1}{1+x^2} = \sum_{k=0}^{\infty} (-1)^k x^{2k} = 1 - x^2 + x^4 - x^6 + \dots$$

converges when $|x| < 1$

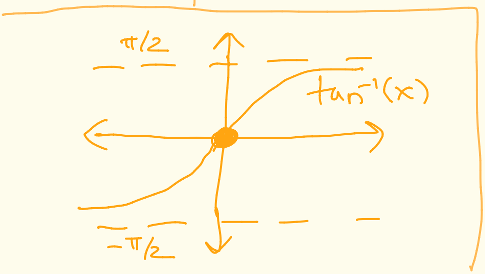
Integrate

$$\underbrace{\int \frac{dx}{1+x^2}}_{\tan^{-1}(x)} = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1} + C = C + x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$$

$$\tan^{-1}(x) = C + \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1} = C + x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$$

To find C, plug in $x = 0$.

$$0 = \tan^{-1}(0) = C + \underbrace{0 - \frac{0^3}{3} + \frac{0^5}{5} - \dots}_{0}$$



So, $C = 0$.

$$\tan^{-1}(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1} = x - x^3/3 + x^5/5 - \dots$$

converges $|x| < 1$, $-1 < x < 1$

Workshop

8.3

(33) Does this series converge or diverge? If it converges what is its sum.

$$\sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k 5^{3-k} = \sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k 5^3 5^{-k}$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k 5^3 \left(\frac{1}{5}\right)^k = \sum_{k=0}^{\infty} \left(\frac{1}{4\cdot 5}\right)^k \cdot 5^3$$

$$= 5^3 \sum_{k=0}^{\infty} \left(\frac{1}{20}\right)^k = 5^3\left[\frac{1}{1-\frac{1}{20}}\right] = 5^3\left[\frac{20}{19}\right] = \frac{125\cdot 20}{19}$$

$$= \frac{2500}{19}$$

The series converges.

$r = \frac{1}{20}$

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r} \quad -1 < r < 1$$

$|r| < 1$