

3/2  
Monday  
Week 7

(7.4)

(47)

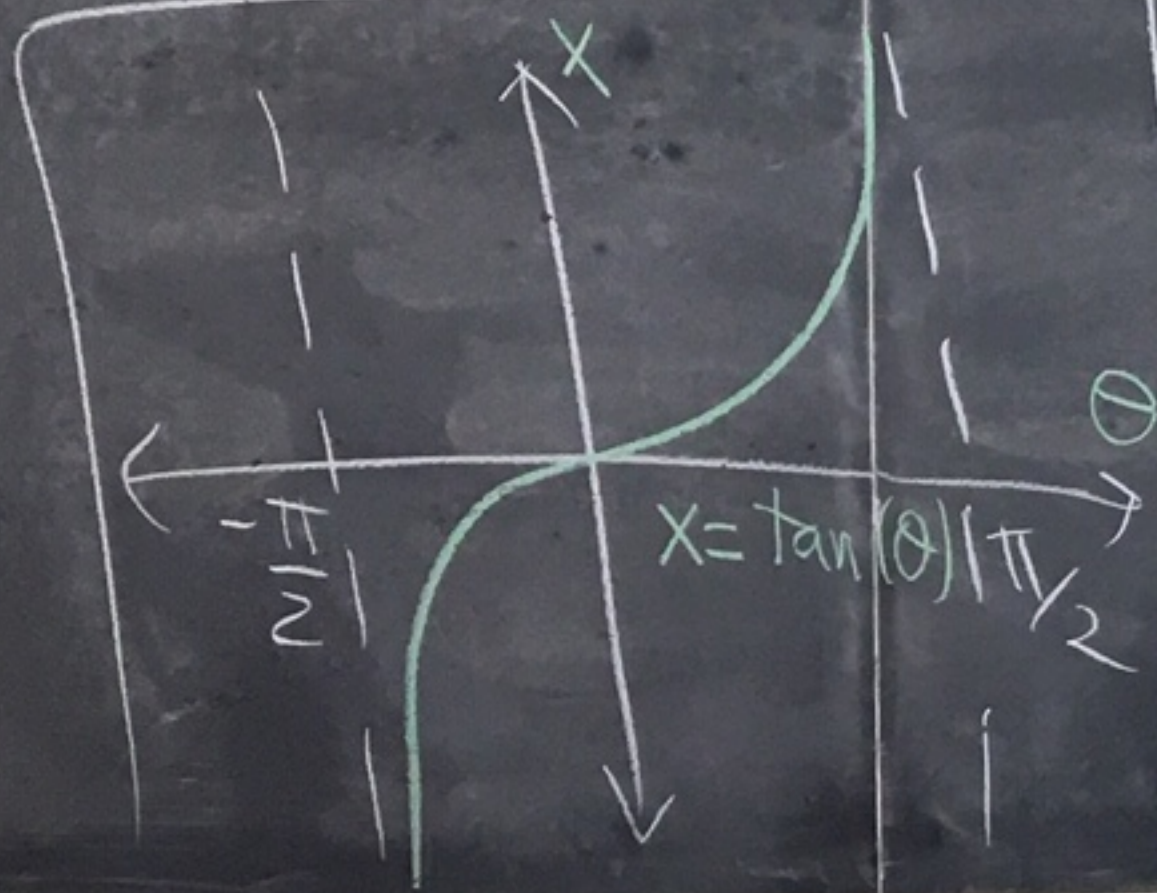
$$\int_0^1 \frac{dx}{\sqrt{x^2+16}}$$

$$\int_0^{\tan^{-1}(\frac{1}{4})} \frac{4 \sec^2(\theta)}{\sqrt{16 \tan^2(\theta)+16}} d\theta = \int_0^{\tan^{-1}(\frac{1}{4})} \frac{4 \sec^2(\theta)}{\sqrt{16} \sqrt{\tan^2(\theta)+1}} d\theta$$

$$= \int_0^{\tan^{-1}(\frac{1}{4})} \frac{\sec^2(\theta)}{\sqrt{\sec^2(\theta)}} d\theta = \int_0^{\tan^{-1}(\frac{1}{4})} \frac{\sec^2(\theta)}{\sec(\theta)} d\theta$$

$$= \int_0^{\tan^{-1}(\frac{1}{4})} \sec(\theta) d\theta$$

$\tan^2(\theta)+1 = \sec^2(\theta)$



When  $x=0$ ,  
 $0 = 4 \tan(\theta)$   
 $0 = \tan(\theta)$   
 $\theta = 0$

When  $x=1$ ,  
 $1 = 4 \tan(\theta)$   
 $\frac{1}{4} = \tan(\theta)$   
 $\tan^{-1}(\frac{1}{4}) = \theta$

$\sqrt{x^2+a^2}$   
 $x = a \tan(\theta)$   
 $x = 4 \tan(\theta)$   
 $dx = 4 \sec^2(\theta) d\theta$

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$\ln |\sec(\theta) + \tan(\theta)| \Big|_{\tan^{-1}(\frac{1}{4})}^0$$

$$= \ln |\sec(\tan^{-1}(\frac{1}{4})) + \tan(\tan^{-1}(\frac{1}{4}))|$$

$$- \ln |\sec(0) + \tan(0)|$$

$\frac{1}{\cos(0)} = 1$

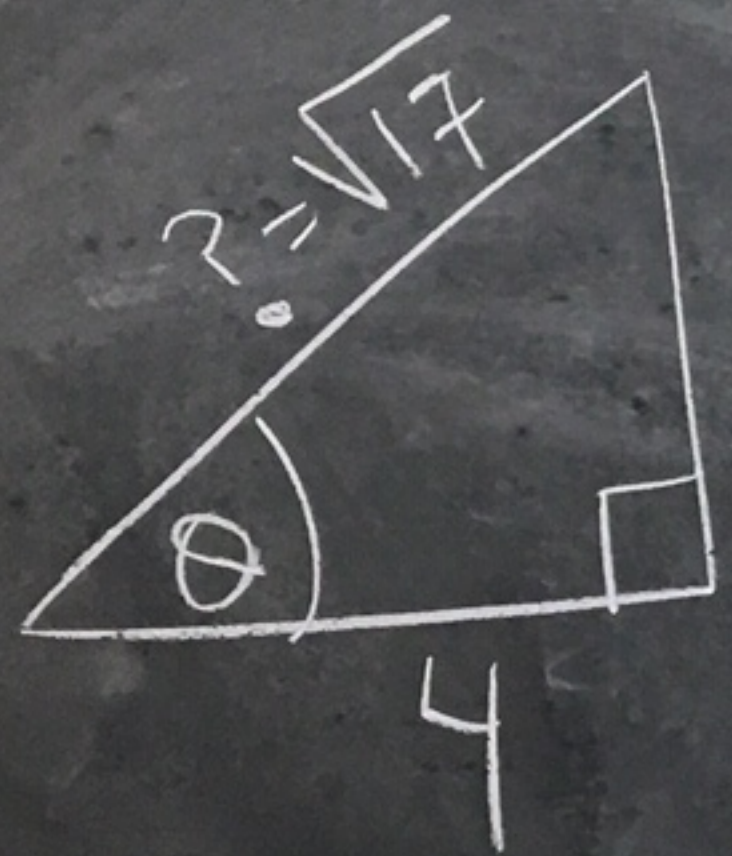
$$\ln(1) = 0$$

$$= \ln \left| \underbrace{\sec(\tan^{-1}(\frac{1}{4}))}_{\sqrt{17}/4} + \underbrace{\tan(\tan^{-1}(\frac{1}{4}))}_{1/4} \right| = \ln \left| \frac{\sqrt{17}}{4} + \frac{1}{4} \right|$$

$$= \ln \left| \frac{\sqrt{17} + 1}{4} \right|$$

Let  $\theta = \tan^{-1}(\frac{1}{4})$

Then  $\tan(\theta) = \frac{1}{4} = \frac{\text{opp}}{\text{adj}}$



$$1^2 + 4^2 = ?^2$$

$$? = \sqrt{17}$$

$$\sec(\tan^{-1}(\frac{1}{4}))$$

$$= \sec(\theta)$$

$$= \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{17}}{4}$$

50 (modified)

$$\int \frac{dx}{x^2 \sqrt{4-x^2}} \xrightarrow{\uparrow} \int \frac{2 \cos(\theta) d\theta}{4 \sin^2(\theta) \sqrt{4-4 \sin^2(\theta)}} \xrightarrow{\uparrow} \int \frac{2 \cos(\theta) d\theta}{4 \sin^2(\theta) \cdot 2 \cos(\theta)}$$

$\sqrt{a^2-x^2}$ $x = a \sin(\theta)$ $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$x = 2 \sin(\theta)$ $dx = 2 \cos(\theta) d\theta$
---	---

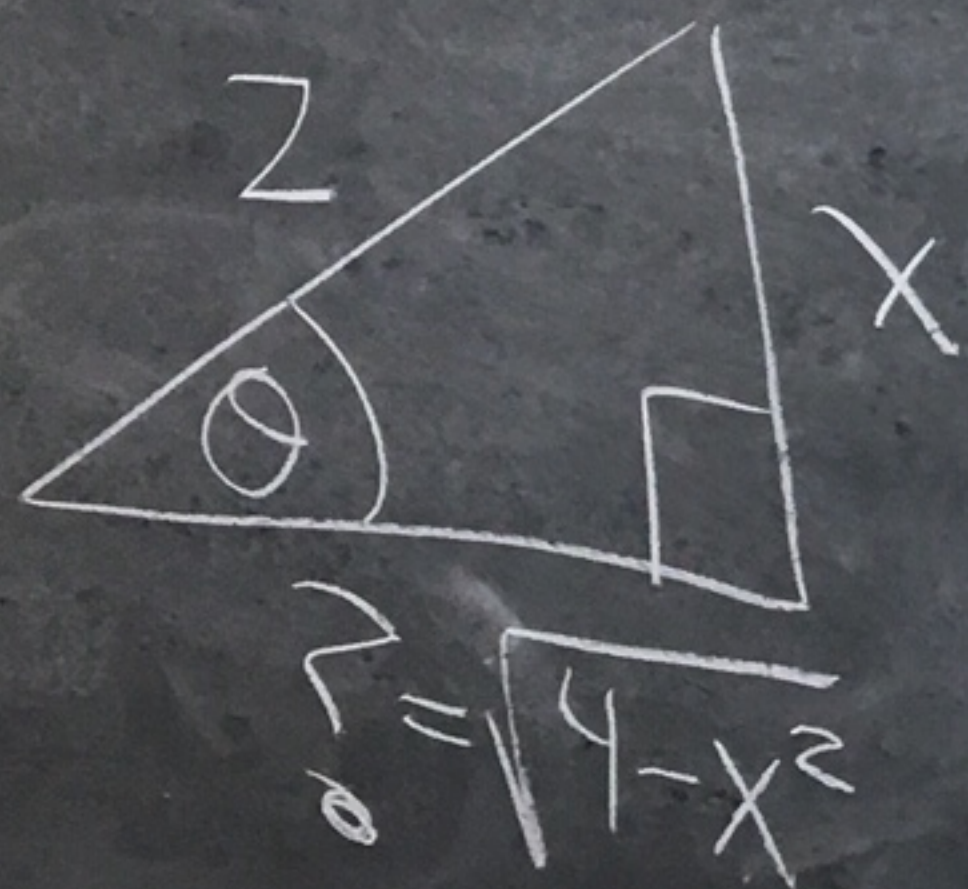
$\sqrt{4-4 \sin^2(\theta)}$ $= \sqrt{4 \sqrt{1-\sin^2(\theta)}}$ $= 2 \sqrt{\cos^2(\theta)}$ $= 2 \cos(\theta)$
--

$$= \frac{1}{4} \int \frac{d\theta}{\sin^2(\theta)}$$
$$= \frac{1}{4} \int \csc^2(\theta) d\theta$$

$$= -\frac{1}{4} \cot(\theta) + C \quad \xrightarrow{\quad} \quad -\frac{1}{4} \frac{\sqrt{4-x^2}}{x} + C$$

$$x = 2 \sin(\theta)$$

$$\sin(\theta) = \frac{x}{2} = \frac{\text{opp}}{\text{hyp}}$$



$$2^2 = x^2 + \text{adj}^2$$

$$\text{adj}^2 = 4 - x^2$$

$$\text{adj} = \sqrt{4-x^2}$$

$$\cot(\theta) = \frac{\text{adj}}{\text{opp}}$$

$$= \frac{\sqrt{4-x^2}}{x}$$

7.8

Does  $\int_{-3}^1 \frac{dx}{(2x+6)^{2/3}}$  converge or diverge? If it converges, what does it converge to?

388

$$\int_{-3}^1 \frac{dx}{(2x+6)^{2/3}}$$

$\frac{1}{(2x+6)^{2/3}}$  is  
 undefined at  
 $x = -3$   
 since  
 $\frac{1}{(2(-3)+6)^{2/3}} = \frac{1}{0^{2/3}}$

diverge? If it converges, what does it converge to?

$$\lim_{t \rightarrow -3^+} \int_t^1 (2x+6)^{-2/3} dx$$

$$\lim_{t \rightarrow -3^+} \left[ \frac{3}{2} (2x+6)^{1/3} \right]_t^1$$

$$\int (2x+6)^{-2/3} dx = \frac{1}{2} \int u^{-2/3} du$$

$$= \frac{1}{2} \frac{u^{1/3}}{(1/3)} + C$$

$$= \frac{3}{2} u^{1/3} + C = \frac{3}{2} (2x+6)^{1/3} + C$$

$u = 2x+6$   
 $du = 2dx$   
 $\frac{1}{2} du = dx$

$$= \lim_{t \rightarrow -3^+} \left[ \frac{3}{2} (2(1)+6)^{1/3} - \frac{3}{2} (2t+6)^{1/3} \right]$$

$$= \frac{3}{2} (8)^{1/3} - \frac{3}{2} (2(-3)+6)^{1/3}$$

$$= \frac{3}{2} \cdot 2 - 0 = \boxed{3}$$

So,  $\int_{-3}^1 \frac{dx}{(2x+6)^{3/2}}$  converges to 3.

7.8

14

$$\int_2^{\infty} \frac{dy}{y \ln(y)} = \lim_{t \rightarrow \infty} \int_2^t \frac{dy}{y \ln(y)} = \lim_{t \rightarrow \infty} \int_2^t \frac{dy}{y \ln(y)}$$

$\uparrow$

$$\int \frac{dy}{y \ln(y)} = \int \frac{1}{u} du = \ln|u| + C$$

$$\begin{aligned} u &= \ln(y) \\ du &= \frac{1}{y} dy \end{aligned}$$

$$= \ln|\ln(y)| + C$$



$$= \lim_{t \rightarrow \infty} \ln(\ln(y)) \Big|_z^t = \lim_{t \rightarrow \infty} \left[ \underbrace{\ln(\ln(t))}_{\infty} - \underbrace{\ln(\ln(z))}_{\#} \right] = \infty$$

So,  $\int_z^{\infty} \frac{dy}{y \ln(y)}$  diverges.

7.5

17

$$\int_{-1}^2 \frac{5x}{x^2 - x - 6} dx$$

$$\frac{5x}{x^2 - x - 6} = \frac{5x}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$5x = A(x+2) + B(x-3)$$

$$\boxed{x=3}$$

$$5(3) = A(5) + B(0)$$

$$\boxed{3 = A}$$

$$\boxed{x=-2}$$

$$5(-2) = A(0) + B(-5)$$

$$\boxed{-2 = B}$$

$$\int \frac{dx}{ax+b} = \frac{1}{a} \ln|ax+b| + C$$

$$\int_{-1}^2 \frac{5x}{x^2-x-6} dx$$

$$= \int_{-1}^2 \left[ \frac{3}{x-3} + \frac{2}{x+2} \right] dx$$

$$= \left( 3 \ln|x-3| + 2 \ln|x+2| \right) \Big|_{-1}^2$$

$$= \left[ 3 \ln|2-3| + 2 \ln|2+2| \right. \\ \left. - (3 \ln|-1-3| + 2 \ln|-1+2|) \right]$$

$$= 3 \ln(1) + 2 \ln(4)$$

$$- 3 \ln(4) - 2 \ln(1) = -\ln(4) = \ln(4^{-1}) = \ln\left(\frac{1}{4}\right)$$

$$\textcircled{7.3} \int \cos^4(\theta) d\theta = \int (\cos^2(\theta))^2 d\theta$$

$$= \int \left( \frac{1}{2} + \frac{1}{2} \cos(2\theta) \right)^2 d\theta = \int \left[ \frac{1}{4} + 2 \cdot \frac{1}{4} \cos(2\theta) + \frac{1}{4} \underbrace{\cos^2(2\theta)}_{\frac{1}{2} + \frac{1}{2} \cos(4\theta)} \right] d\theta$$

$$\boxed{\cos^2(\theta) = \frac{1}{2} + \frac{1}{2} \cos(2\theta)}$$

$$= \int \left[ \frac{1}{4} + \frac{1}{2} \cos(2\theta) + \frac{1}{8} + \frac{1}{8} \cos(4\theta) \right] d\theta$$

$$= \frac{1}{4}\theta + \frac{1}{2} \cdot \frac{1}{2} \sin(2\theta) + \frac{1}{8}\theta + \frac{1}{8} \cdot \frac{1}{4} \sin(4\theta) + C$$

$$= \frac{3}{8}\theta + \frac{1}{4} \sin(2\theta) + \frac{1}{32} \sin(4\theta) + C$$

7.3)

$$\int \sin^{11}(x) \cos^{10}(x) dx = \int \sin^{10}(x) \cos^{10}(x) \sin(x) dx$$

$$= \int (\sin^2(x))^5 \cos^{10}(x) \sin(x) dx = \int (1 - \cos^2(x))^5 \cos^{10}(x) \sin(x) dx$$

$$= \int (1 - u^2)^5 u^{10} du = - \left[ 1^5 + 5 \cdot 1^4 \cdot (-u^2) + 10 \cdot 1^3 \cdot (-u^2)^2 + 10 \cdot 1^2 \cdot (-u^2)^3 + 5 \cdot 1 \cdot (-u^2)^4 + 1 \cdot (-u^2)^5 \right] \cdot u^{10} du$$

$$u = \cos(x)$$

$$du = -\sin(x) dx$$

$$-du = \sin(x) dx$$

$$(a+b)^5 = 1 \cdot a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1 \cdot b^5$$

$$\begin{array}{cccccccc}
 & & & & & & & & & & \\
 & & & & & & & & & & 1 \\
 & & & & & & & & & & 1 & \\
 & & & & & & & & & & 1 & 2 & 1 \\
 & & & & & & & & & & 1 & 3 & 3 & 1 \\
 & & & & & & & & & & 1 & 4 & 6 & 4 & 1 \\
 & & & & & & & & & & 1 & 5 & 10 & 10 & 5 & 1 \\
 & & & & & & & & & & 1 & 6 & 15 & 20 & 15 & 6 & 1 \\
 & & & & & & & & & & 1 & 7 & 21 & 35 & 35 & 21 & 7 & 1 \\
 & & & & & & & & & & 1 & 8 & 28 & 56 & 70 & 56 & 28 & 8 & 1 \\
 & & & & & & & & & & 1 & 9 & 36 & 84 & 126 & 126 & 84 & 36 & 9 & 1 \\
 & & & & & & & & & & 1 & 10 & 45 & 120 & 210 & 252 & 210 & 120 & 45 & 10 & 1
 \end{array}$$

$$1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$$

$$10a^2b^3 + 5ab^4 + 1 \cdot b^5$$



$$= - \int \left[ u^{10} - 5u^{12} + 10u^{14} - 10u^{16} + 5u^{18} - u^{20} \right] du$$

$$= - \frac{u^{11}}{11} + 5 \frac{u^{13}}{13} - 10 \frac{u^{15}}{15} + 10 \frac{u^{17}}{17} - 5 \frac{u^{19}}{19} + \frac{u^{21}}{21} + C$$

$$= - \frac{1}{11} \cos^{11}(x) + \frac{5}{13} \cos^{13}(x) - \frac{2}{3} \cos^{15}(x) + \frac{10}{17} \cos^{17}(x) - \frac{5}{19} \cos^{19}(x) + \frac{1}{21} \cos^{21}(x) + C$$

sin<sup>11</sup>

7.2

22

$$\int \overset{dv}{x} \cdot \overset{u}{\tan^{-1}(x^2)} dx$$

$$= \underset{uv}{\frac{1}{2} x^2 \tan^{-1}(x^2)} - \int \frac{2x}{1+x^4} \cdot \frac{1}{2} x^2 dx$$

$$\int v du$$

LIATE

$$u = \tan^{-1}(x^2) \quad du = \frac{1}{1+(x^2)^2} \cdot (2x) dx$$

$$dv = x dx \quad v = \frac{1}{2} x^2$$

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\frac{2x}{1+x^4} \cdot \frac{1}{2} x^2 dx$$

$\int v du$

$$\frac{1}{2} x^2 + \tan^{-1}(x^2) - \int \frac{x^3}{1+x^4} dx$$

$$\frac{1}{2} x^2 + \tan^{-1}(x^2) - \frac{1}{4} \int \frac{1}{u} du$$

$$= \frac{1}{2} x^2 + \tan^{-1}(x^2) - \frac{1}{4} \ln|u| + C$$

$$= \frac{1}{2} x^2 + \tan^{-1}(x^2) - \frac{1}{4} \ln|1+x^4| + C$$

$$\begin{aligned} u &= 1+x^4 \\ du &= 4x^3 dx \\ \frac{1}{4} du &= x^3 dx \end{aligned}$$

Emergency  
CAL STATE L.A.  
CALIFORNIA STATE UNIVERSITY, LOS ANGELES

**EVACUATION**  
PUBLIC SAFETY  
911  
EMERGENCY SERVICES

**SHELTER IN PLACE**  
EMERGENCY NOTIFICATION

**POWER OUTAGE**

**HAZARDOUS MATERIALS**

**FIRE**