Math 2120

3/24/20

Tuesday

9.4 continued...

Ex: Find the first three terms in the Maclaurin series for $f(x) = e^x \sin(x)$

Taylor series with $a=0$

both converge for all x

$$f(x) = e^x \sin(x)$$

$$= \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)$$

$$= x + x^2 + \underbrace{1\cdot\left(-\frac{x^3}{3!}\right) + \frac{x^2}{2!}\cdot x}_{x^3 \text{ term}} + \cdots$$

$3! = 6,\ 2! = 2$

$$= x + x^2 + \left(-\frac{1}{6} + \frac{1}{2}\right)x^3 + \cdots$$

$$= x + x^2 + \frac{1}{3}x^3 + \cdots$$

Converge for all x

Ex: Same question with $f(x) = \tan(x)$

$$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots}{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots}$$

tan(x)

cos(x)

sin(x)

$$x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \cdots$$

$$1 - \frac{x^2}{2} + \frac{x^4}{24} - \cdots \;\Big)\; x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots$$

$$-\left(x - \frac{x^3}{2} + \frac{x^5}{24} - \cdots\right)$$

$$0 + \left(-\frac{1}{6} + \frac{1}{2}\right)x^3 + \left(\frac{1}{120} - \frac{1}{24}\right)x^5 - \cdots$$

$$\frac{1}{3}x^3 - \frac{1}{30}x^5 - \cdots$$

$$-\left(\frac{1}{3}x^3 - \frac{1}{6}x^5 + \cdots\right)$$

$$0 + \left(-\frac{1}{30} + \frac{1}{6}\right)x^5 + \cdots$$

$$\frac{2}{15}x^5 + \cdots$$

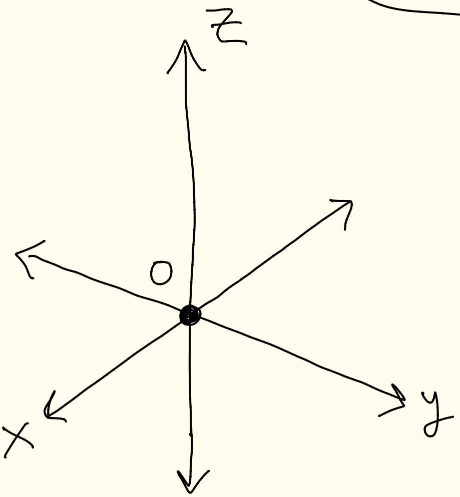
$$-\left(\frac{2}{15}x^5 - \frac{1}{15}x^7 + \cdots\right)$$

$$0 + ?\,x^7 + \cdots$$

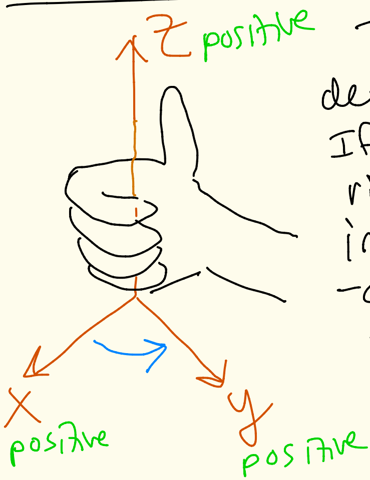
$2! = 2$
$3! = 6$
$4! = 24$
$5! = 120$
$6! = 720$

$$\tan(x) = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \cdots$$

11.1 / 11.2 - Vectors in the plane / Vectors in 3d



To represent 3d space, we choose a fixed point O (the origin) and three directed lines through O that are perpendicular to each other, called the x-axis, y-axis, and z-axis (the coordinate axis).



The direction of the z-axis is determined by the <u>right-hand rule</u>. If you curl the fingers in your right hand around the z-axis in the direction of a 90° counter-clockwise rotation from the positive x-axis to the positive y-axis, then your thumb points in the positive z direction.

We label points in 3d as ordered triples (x,y,z).

plot
$(1,0,0)$
$(-1,2,0)$
$(0,-2,2)$
$(2,3,1)$