Monday
3/9
Week 8

Ratio test revisted

Better ratio test

Let $\sum a_k$ be an infinite series, where $a_k \neq 0$ for large enough k,

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If

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Let

$$L = \lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right|$$

If $0 \leq L < 1$, then
$\sum a_k$ is absolutely convergent.

$\sum a_k$ is <u>absolutely convergent</u>
means: $\sum a_k$ converges
and $\sum |a_k|$ converges

If $L > 1$, then $\sum a_k$ diverges.

If $L = 1$ or the limit L doesn't exist, then the test is inconclusive.

Ex: Find the interval and radius of convergence of

$$\sum_{k=0}^{\infty} \frac{(-1)^k (x-2)^k}{4^k} = \underbrace{1}_{k=0} - \underbrace{\frac{(x-2)}{4}}_{k=1} + \underbrace{\frac{(x-2)^2}{4^2}}_{k=2} - \underbrace{\frac{(x-2)^3}{4^3}}_{k=3} + \cdots$$

$$= 1 - \frac{1}{4}(x-2) + \frac{1}{4^2}(x-2)^2 - \frac{1}{4^3}(x-2)^3 + \cdots$$

power series centered at $a=2$

power serie[s]

$\sum_{k=0}^{\infty} c_k (x-a)^k$

$|ab| = |a||b|$

$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$\left.2)\right)^3$

$3 + \dots$

$= 3$

$+ \dots$

at $a = 2$

$\boxed{\begin{array}{l}\text{power series centered at } a \\ \sum_{k=0}^{\infty} c_k(x-a)^k = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots\end{array}}$

What x's can we plug in that make the series converge?

Ratio test time!

$$L = \lim_{k\to\infty}\left|\frac{\dfrac{(-1)^{k+1}(x-2)^{k+1}}{4^{k+1}}}{\dfrac{(-1)^k(x-2)^k}{4^k}}\right| = \lim_{k\to\infty}\left|\frac{4^k}{(-1)^k(x-2)^k}\cdot\frac{(-1)^{k+1}(x-2)^{k+1}}{4^{k+1}}\right|$$

$$\boxed{\begin{array}{l}|ab| = |a||b| \\ \left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}\end{array}}$$

$$= \lim_{k\to\infty}\left|\frac{(-1)(x-2)}{4}\right| = \left|\frac{(-1)(x-2)}{4}\right| = \frac{|(-1)||x-2|}{|4|} = \frac{|x-2|}{4}$$

We get absolute convergence when $L < 1$.

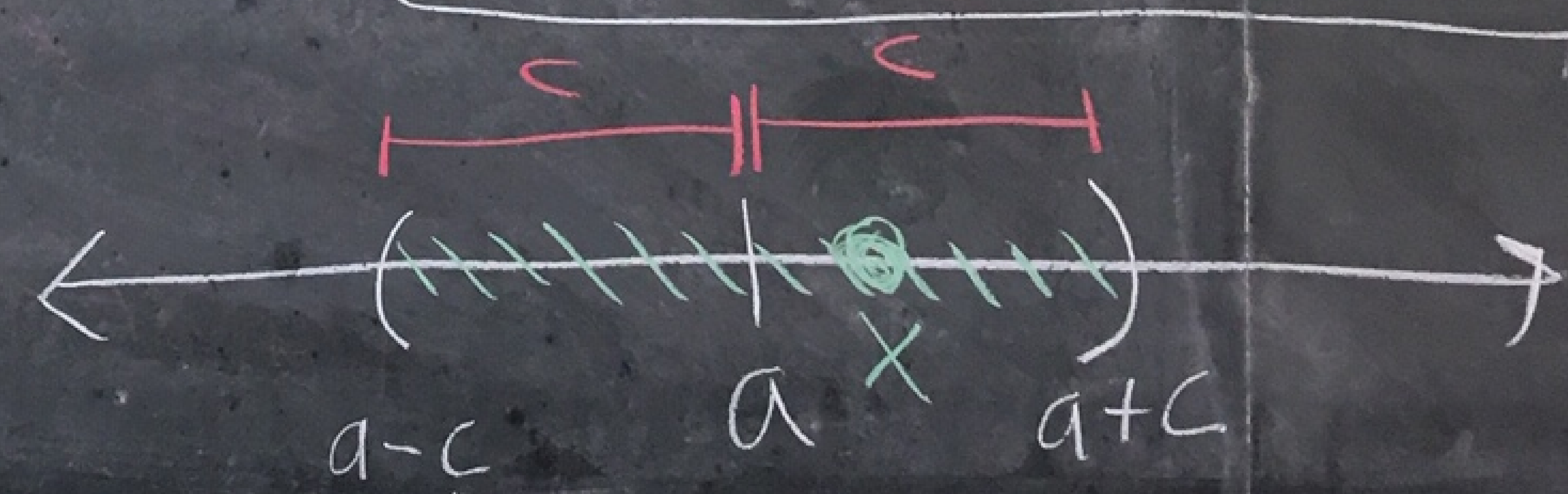
That is when $\dfrac{|x-2|}{4} < 1$

That is when $|x-2| < 4$.

Absolute value ideas ($c > 0$)

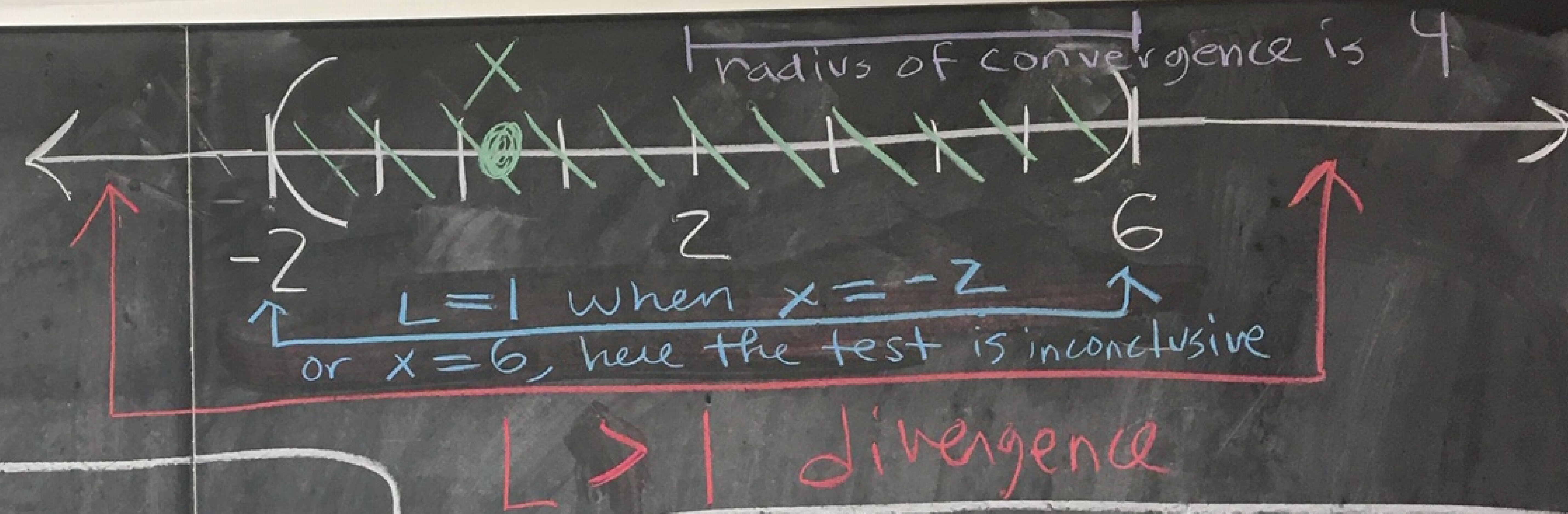
$|x-a| < c$ ← the distance between x and a is less than c

$a-c < x < a+c$



$2-4 < x < 2+4$

$\boxed{-2 < x < 6}$



test the endpoints

$\boxed{x=-2}$: $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k (x-2)^k}{4^k} = \sum_{k=0}^{\infty} \frac{(-1)^k(-2-2)^k}{4^k} = \sum_{k=0}^{\infty} \frac{(-1)^k(-4)^k}{4^k} = \sum_{k=0}^{\infty} \frac{(-1)^k(-1)^k 4^k}{4^k}$

$\displaystyle = \sum_{k=0}^{\infty} (-1)^{2k} = \sum_{k=0}^{\infty} 1 = 1+1+1+1+\cdots$ diverges

$\boxed{x=6}$:

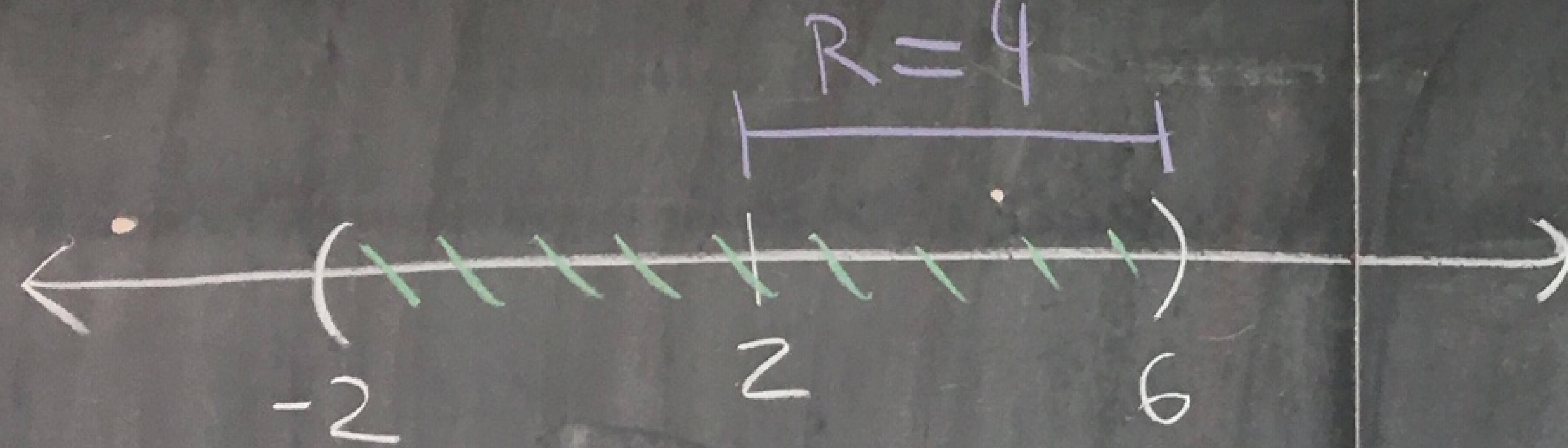
$$\sum_{k=0}^{\infty} \frac{(-1)^k (x-2)^k}{4^k} = \sum_{k=0}^{\infty} \frac{(-1)^k (6-2)^k}{4^k}$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k 4^k}{4^k} = \sum_{k=0}^{\infty} (-1)^k = 1-1+1-1+1-1+\cdots$$

By the divergence since $\lim_{k\to\infty} (-1)^k \neq 0$

the series diverges.

Interval of convergence



interval of convergence: $(-2,6)$ or $-2<x<6$

radius of convergence: $R=4$