Math 2120

4/14/20

Let's go to chapter 10 and work on that for a little bit. Then we will come back to the rest of chapter 11.

10.1 — Parametric Equations (in 2D)

Suppose that x and y are both given as functions of a third variable t (called a <u>parameter</u>) by some equations

$$x = f(t), \quad y = g(t) \longleftarrow \left(\underline{\text{parametric equations}}\right)$$

Each value of t determines a point (x, y). As t varies, the point $(x, y) = (f(t), g(t))$ varies and traces out a curve, which we call a <u>parametric curve</u>.

Ex: Sketch the parametric curve given by

$$x = t^2 - 2t, \quad y = t + 1$$

t	x	y
-2	8	-1
-1	3	0
0	0	1
1	-1	2
2	0	3
3	3	4
4	8	5

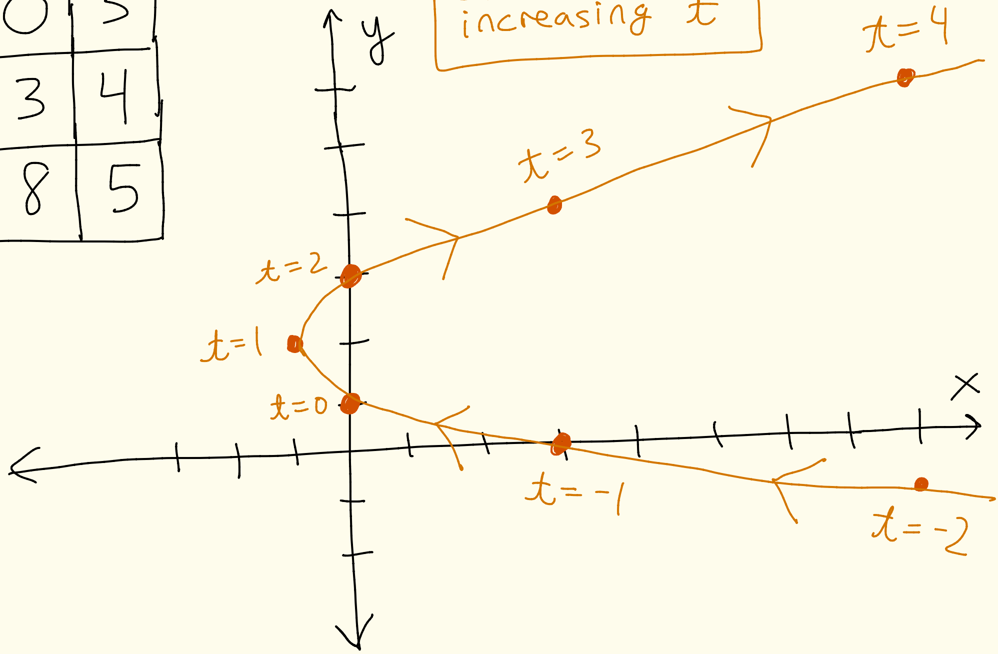
You can also "eliminate" t:

$y = t + 1 \longrightarrow t = y - 1$

$\longrightarrow x = (y-1)^2 - 2(y-1)$ $\longleftarrow$ $\boxed{x = t - 2t}$

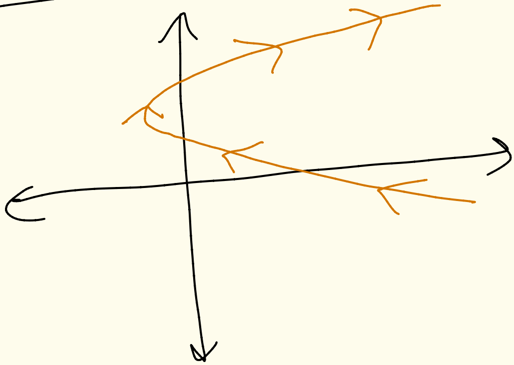
$\longrightarrow \boxed{x = y^2 - 4y + 3}$

arrow is in the direction of increasing t



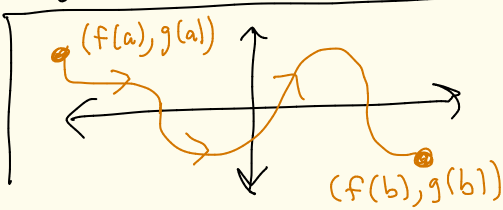
The direction in which the curve is generated as the parameter t increases is called the <u>positive orientation</u> of the curve.

<u>last example</u>



positive orientation (direction as t increases)

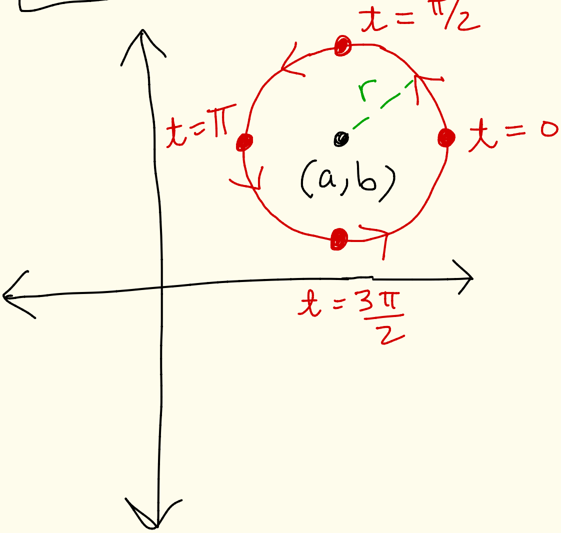
The curve with parametric equations
$x=f(t)$, $y=g(t)$, $a \le t \le b$
has <u>initial point</u> $(x,y)=(f(a),g(a))$
and <u>terminal point</u>
$(x,y)=(f(b),g(b))$



A parametric set of equations for a circle of radius r centered at (a,b) is:

(counterclockwise direction)

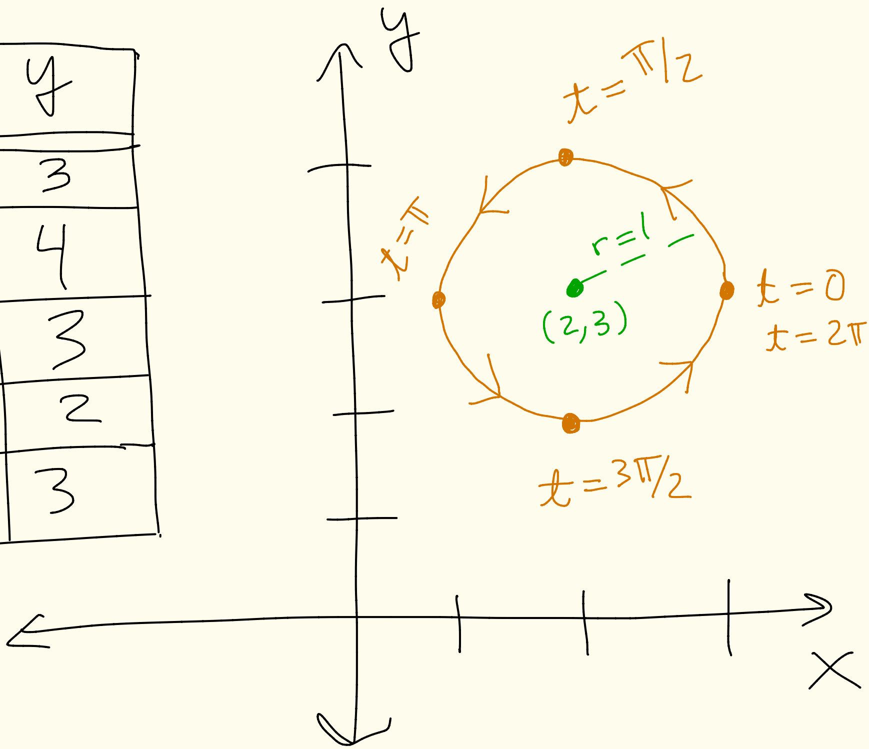
$$\boxed{\begin{aligned} x &= a + r\cos(t) \qquad 0 \le t \le 2\pi \\ y &= b + r\sin(t) \end{aligned}}$$



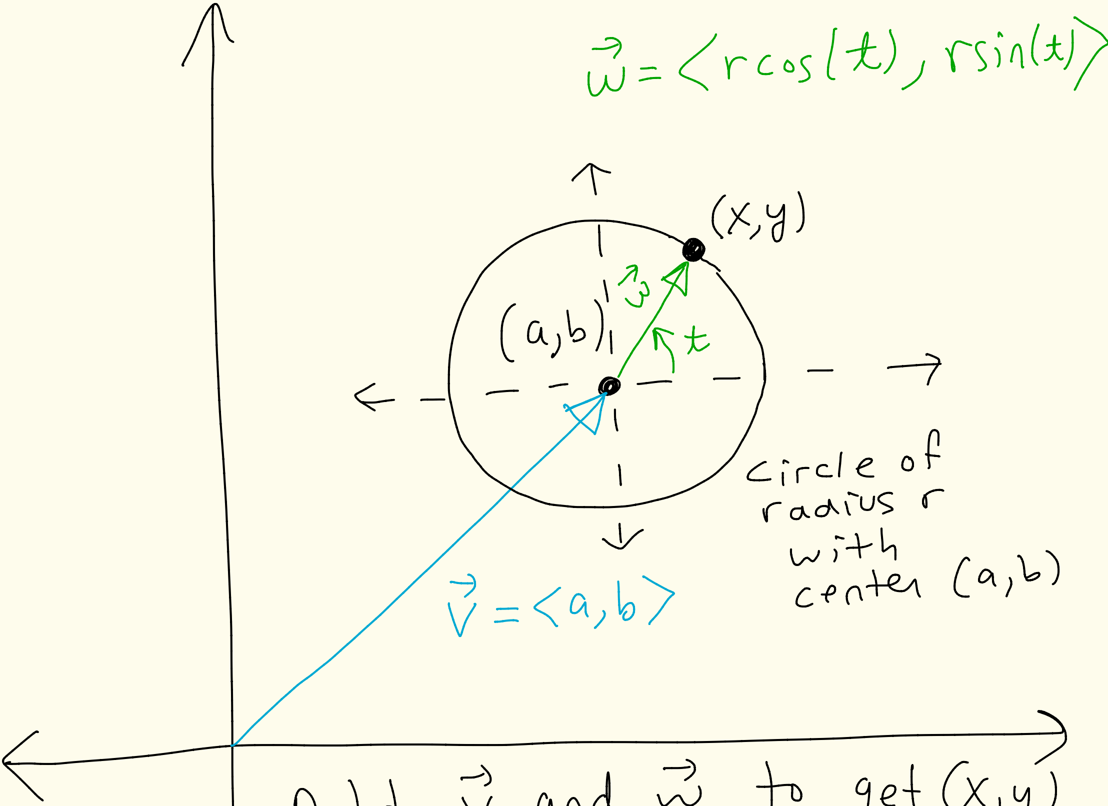
Ex: Circle with center $(2,3)$ and radius 1.

$$x = 2 + 1\cdot\cos(t) \qquad 0 \le t \le 2\pi$$
$$y = 3 + 1\cdot\sin(t)$$

t	x	y
0	3	3
$\pi/2$	2	4
π	1	3
$3\pi/2$	2	2
2π	3	3



Why this formula works



Add $\vec{v}$ and $\vec{w}$ to get (x,y).

$$\vec{v} + \vec{w} = \langle a,b\rangle + \langle r\cos(t), r\sin(t)\rangle$$
$$= \langle a + r\cos(t), b + r\sin(t)\rangle$$
$$= \langle x, y\rangle$$

$$x = a + r\cos(t), \quad y = b + r\sin(t)$$