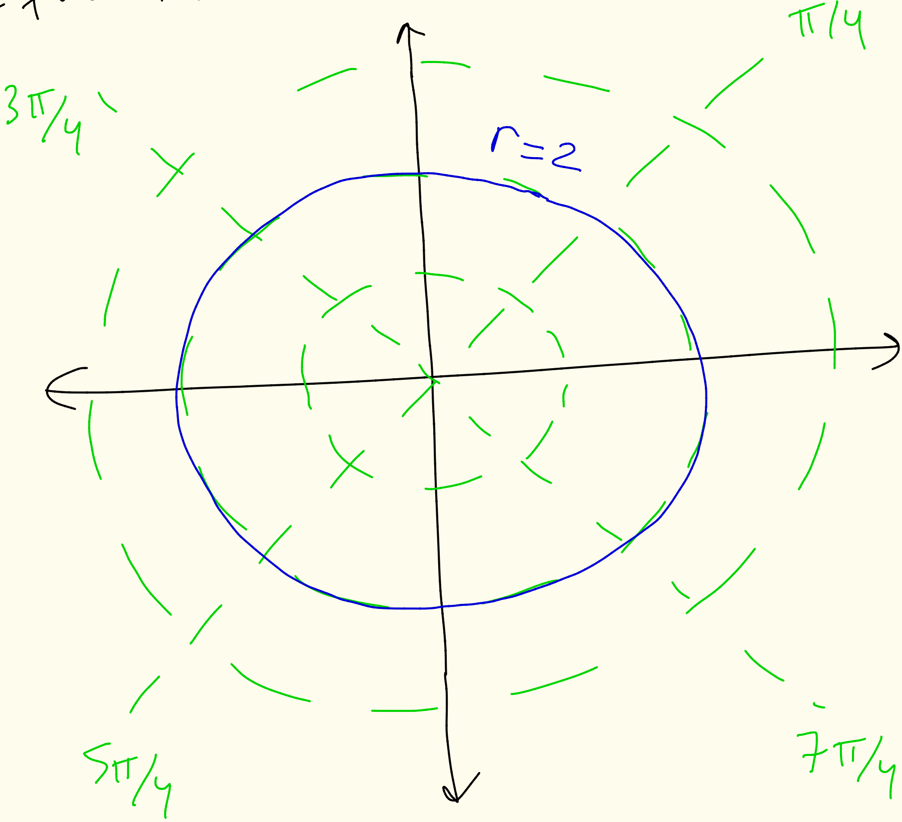
Math 2120

4/20/20

week 13

Ex: Graph the polar equation $r = 2$.



Its every point that satisfies $\underbrace{r = 2}_{r \text{ is distance from origin,}}$. θ can be anything.

r is distance from origin, This is $x^2 + y^2 = 4$.

Ex: Plot the polar equation $\theta = \pi/4$.

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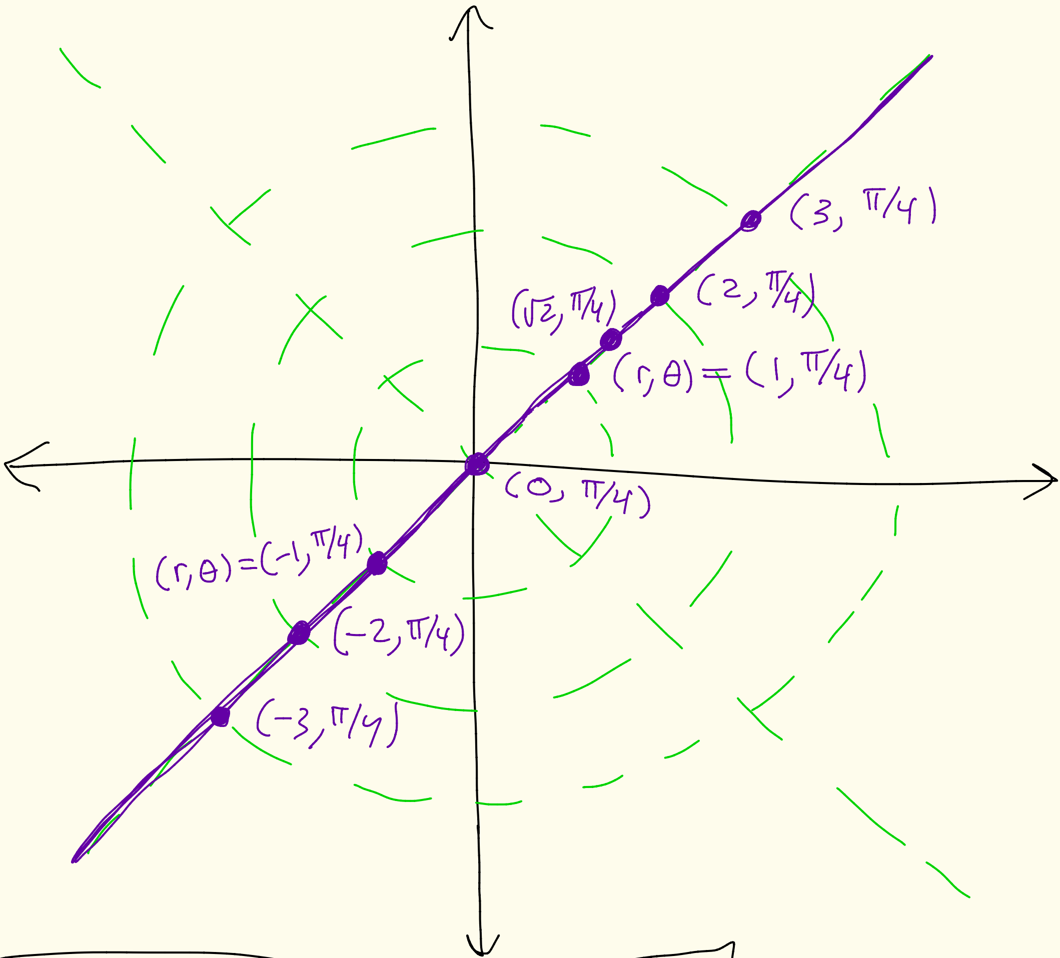
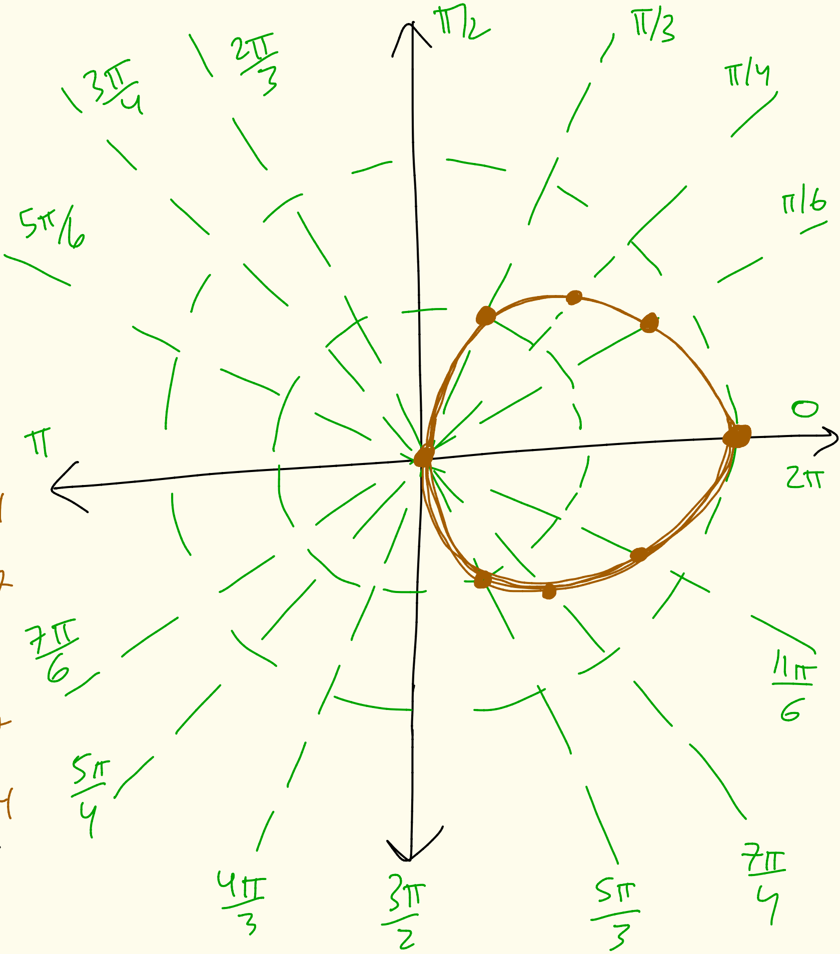


$\boxed{\theta = \pi/4 \text{ is the } y = x \text{ line}}$

Ex: Graph $r = 2\cos(\theta)$

θ	$r = 2\cos(\theta)$
0	$2(1) = 2$
$\pi/6$	$2\left(\frac{\sqrt{3}}{2}\right) \approx 1.7$
$\pi/4$	$2\left(\frac{\sqrt{2}}{2}\right) \approx 1.4$
$\pi/3$	$2\left(\frac{1}{2}\right) = 1$
$\pi/2$	$2(0) = 0$
$\frac{2\pi}{3}$	$2\left(-\frac{1}{2}\right) = -1$
$\frac{3\pi}{4}$	$2\left(-\frac{\sqrt{2}}{2}\right) \approx -1.4$
$\frac{5\pi}{6}$	$2\left(-\frac{\sqrt{3}}{2}\right) \approx -1.7$
π	$2(-1) = -2$
$\frac{7\pi}{6}$	$2\left(-\frac{\sqrt{3}}{2}\right) \approx -1.7$
$\frac{5\pi}{4}$	$2\left(-\frac{\sqrt{2}}{2}\right) \approx -1.4$
$\frac{4\pi}{3}$	$2\left(-\frac{1}{2}\right) = -1$
$\frac{3\pi}{2}$	$2(0) = 0$
$5\pi/3$	$2\left(\frac{1}{2}\right) = 1$
$\frac{7\pi}{4}$	$2\left(\frac{\sqrt{2}}{2}\right) \approx 1.4$
$\frac{11\pi}{6}$	$2\left(\frac{\sqrt{3}}{2}\right) \approx 1.7$
2π	$2(1) = 2$

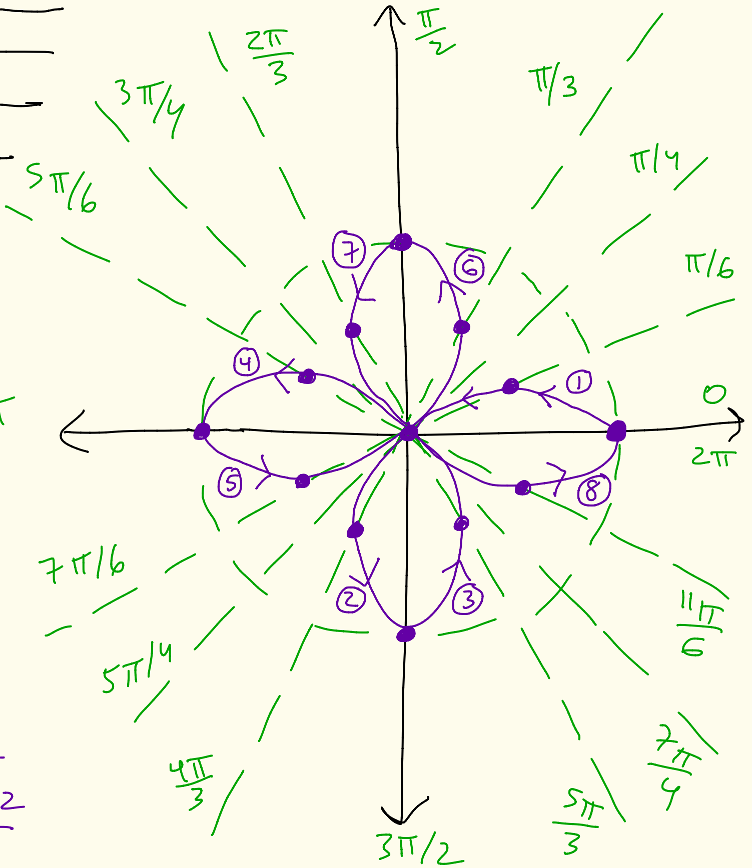


$$\boxed{\begin{array}{l}\sqrt{2} \approx 1.4 \\ \sqrt{3} \approx 1.7\end{array}}$$

we go around the shape twice, in the counterclockwise direction

Ex: Sketch $r = \cos(2\theta)$

θ	$r = \cos(2\theta)$
0	1
$\pi/6$	$\cos(2\cdot\frac{\pi}{6}) = 1/2$
$\pi/4$	$\cos(2\cdot\frac{\pi}{4}) = 0$
$\pi/3$	$\cos(2\cdot\pi/3) = -1/2$
$\pi/2$	$\cos(2\cdot\frac{\pi}{2}) = -1$
$2\pi/3$	$\cos(2\cdot\frac{2\pi}{3}) = -\frac{1}{2}$
$3\pi/4$	$\cos(2\cdot\frac{3\pi}{4}) = 0$
$5\pi/6$	$\cos(2\cdot\frac{5\pi}{6}) = \frac{1}{2}$
π	$\cos(2\cdot\pi) = 1$
$7\pi/6$	$\cos(2\cdot\frac{7\pi}{6}) = \frac{1}{2}$
$5\pi/4$	$\cos(2\cdot\frac{5\pi}{4}) = 0$
$4\pi/3$	$\cos(2\cdot\frac{4\pi}{3}) = -\frac{1}{2}$
$3\pi/2$	$\cos(2\cdot\frac{3\pi}{2}) = -1$
$5\pi/3$	$\cos(2\cdot\frac{5\pi}{3}) = -1/2$
$7\pi/4$	$\cos(2\cdot\frac{7\pi}{4}) = 0$
$11\pi/6$	$\cos(2\cdot\frac{11\pi}{6}) = 1/2$
2π	$\cos(2\cdot 2\pi) = 1$



$\frac{14\pi}{6} = \frac{7\pi}{3} = 2\pi + \frac{\pi}{3}$

$\frac{10\pi}{3} = 2\pi + \frac{4\pi}{3}$

$\frac{8\pi}{3} = 2\pi + \frac{2\pi}{3}$

$\frac{22\pi}{6} = \frac{11\pi}{3} = 2\pi + \frac{5\pi}{3}$

Calculus Workshop

10.1

(5) Find parametric equations for $y = x^2$.

$x = t$

$y = t^2$

t can be any real number

<u>Ex:</u> $y = x^3 + x - 5$

$x = t$

$y = t^3 + t - 5$

t any real number

$\boxed{9.2}$

21 $\displaystyle\sum_{k=1}^{\infty} \frac{k^2 x^{2k}}{k!}$

Find the interval of convergence.

Use the ratio test.

$$L = \lim_{k\to\infty} \left| \frac{(k+1)^2 x^{2(k+1)}}{(k+1)!} \cdot \frac{k!}{k^2 x^{2k}} \right|$$

$$= \lim_{k\to\infty} \left| \frac{(k+1)^{\cancel{2}\,1} \cancel{x^{2k}} x^2}{\cancel{(k+1)} \cancel{k!}} \cdot \frac{\cancel{k!}}{k^2 \cancel{x^{2k}}} \right|$$

$$= \lim_{k\to\infty} |x^2| \left| \frac{k+1}{k^2} \right| = |x^2| \lim_{k\to\infty} \frac{k+1}{k^2}$$

$$= |x^2| \cdot 0 = 0 < 1$$

So, $\displaystyle\sum_{k=1}^{\infty} \frac{k^2 x^{2k}}{k!}$ converges for all x.

$(-\infty, \infty)$ interval of convergence

9.2

34 Get a power series for $\frac{1}{1+4x}$

$$\frac{1}{1+4x} = \frac{1}{1-(-4x)}$$

$$= \sum_{k=0}^{\infty} (-4x)^k = \sum_{k=0}^{\infty} (-1)^k 4^k x^k$$

when

$-\frac{1}{4} < x < \frac{1}{4}$

$$\boxed{\begin{array}{l} r = -4x \\ \text{converge when} \\ |-4x| < 1 \\ 4|x| < 1 \\ |x| < \frac{1}{4} \\ -\frac{1}{4} < x < \frac{1}{4} \end{array}}$$

$$\boxed{\begin{array}{c} \frac{1}{1-r} = \sum_{k=0}^{\infty} r^k \\ |r| < 1 \end{array}}$$

9.2

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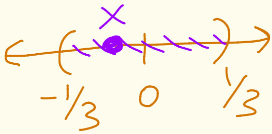
(45) Find a power series and interval of convergence for $f(x) = \ln(1-3x)$.

$$\frac{1}{1-r} = \sum_{k=0}^{\infty} r^k \qquad |r|<1$$

$$\frac{1}{1-3x} = \sum_{k=0}^{\infty} (3x)^k = \sum_{k=0}^{\infty} 3^k x^k$$

$$|3x| < 1 \qquad |x| < \frac{1}{3}$$

$$3|x| < 1 \qquad -\frac{1}{3} < x < \frac{1}{3}$$



$$\int \frac{1}{1-3x}\,dx = \int \sum_{k=0}^{\infty} 3^k x^k \, dx = \underbrace{\left(\sum_{k=0}^{\infty} 3^k \frac{x^{k+1}}{k+1}\right)}_{\text{defenitely converges on } (-1/3,\ 1/3) \text{ might also converge at endpoints}} + C$$

$$\ln(1-3x) = \sum_{k=0}^{\infty} \frac{3^k}{k+1} x^{k+1} + C$$

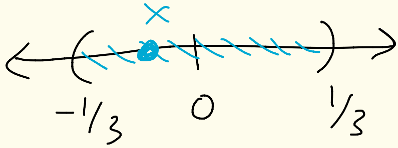
Plug in $x=0$:

$$0 = \ln(1) = \underbrace{\sum \frac{3^k}{k+1} 0^{k+1}}_{0} + C = C$$

So, $$\ln(1-3x) = \sum_{k=0}^{\infty} \frac{3^k}{k+1} x^k$$

$$\ln(1-3x) = \sum_{k=0}^{\infty} \frac{3^k}{k+1} x^k$$

converges for sure on:



What about at the endpoints?

<u>$x = -\frac{1}{3}$:</u>

$$\sum_{k=0}^{\infty} \frac{3^k}{k+1} \left(-\frac{1}{3}\right)^k = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+1}$$

$$(-1)^k \left(\frac{1}{3}\right)^k$$

converges by alt. series test

<u>$x = \frac{1}{3}$:</u>

$$\sum_{k=0}^{\infty} \frac{3^k}{k+1} \left(\frac{1}{3}\right)^k = \sum_{k=0}^{\infty} \frac{1}{k+1}$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots$$

diverges harmonic series

So, $$\ln(1-3x) = \sum_{k=0}^{\infty} \frac{3^k}{k+1} x^k$$

converges when $-\frac{1}{3} \le x < \frac{1}{3}$