Math 2120

4/7/20

Week 11

[Test 2]

pg1

① limits of sequences

② Adding up series: Geometric & telescopic

③ tests: divergence test
integral test
Comparison / limit comparison test
alternating series test
ratio test
p-series

11.3 - The dot product

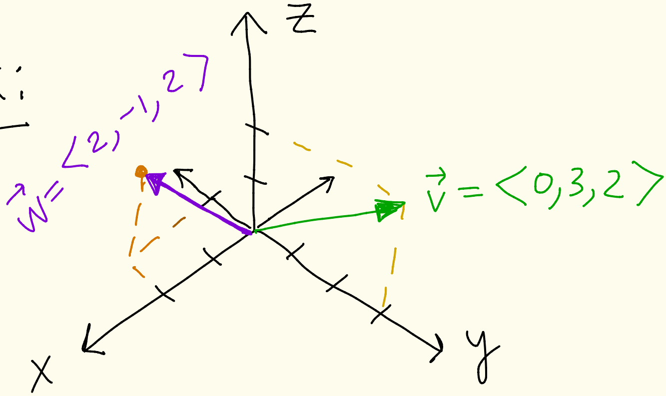
$\boxed{2D}$ Given $\vec{v} = \langle x_1, y_1 \rangle$ and $\vec{w} = \langle x_2, y_2 \rangle$ is the dot product of $\vec{v}$ and $\vec{w}$ is

$$\vec{v} \cdot \vec{w} = x_1 x_2 + y_1 y_2$$

$\boxed{3D}$ Given $\vec{v} = \langle x_1, y_1, z_1 \rangle$ and $\vec{w} = \langle x_2, y_2, z_2 \rangle$ is the dot product of $\vec{v}$ and $\vec{w}$ is

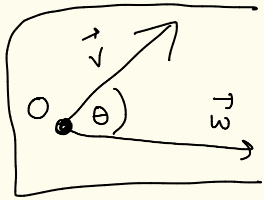
$$\vec{v} \cdot \vec{w} = x_1 x_2 + y_1 y_2 + z_1 z_2$$

Ex:



$$\vec{v} \cdot \vec{w} = 2 \cdot 0 + (-1) \cdot (3) + 2 \cdot 2 = 1$$

Theorem: If θ is the angle between $\vec{v}$ and $\vec{w}$, then



$$\vec{v} \cdot \vec{w} = |\vec{v}||\vec{w}| \cos(\theta)$$

where θ is the angle between the representations of $\vec{a}$ and $\vec{b}$ that start at the origin, where $0 \le \theta \le \pi$.

Ex: Find the angle between $\vec{v} = \langle 0, 3, 2 \rangle$ and $\vec{w} = \langle 2, -1, 2 \rangle$

$\vec{v} \cdot \vec{w} = 1$ (from above), $|\vec{v}| = \sqrt{0^2 + 3^2 + 2^2} = \sqrt{13}$

$|\vec{w}| = \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{9} = 3$

$$\cos(\theta) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}||\vec{w}|} = \frac{1}{3\sqrt{13}} \approx 0.092450032704205\ldots$$

$\theta \approx \cos^{-1}(0.0924500327\ldots)$

$\approx 84.6954\ldots$ degrees

How could

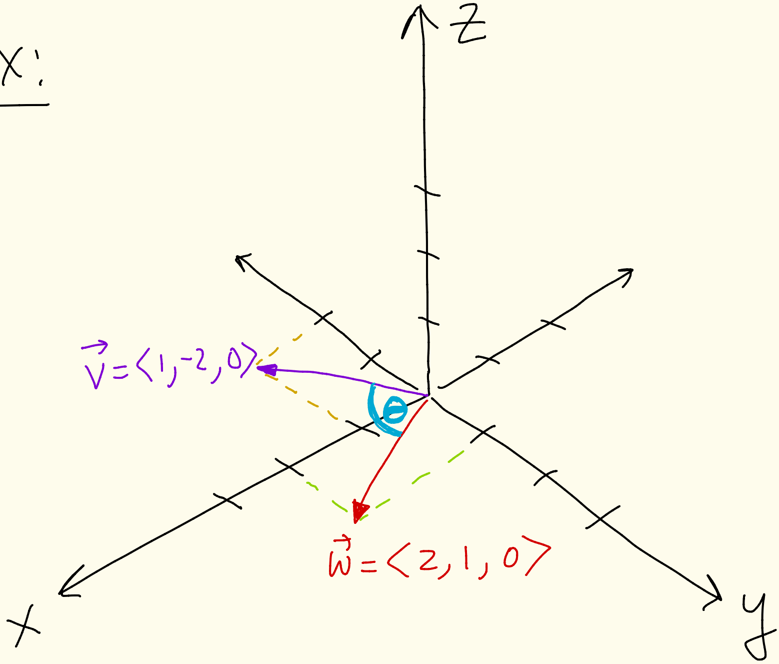
$$\vec{v}\cdot\vec{w} = |\vec{v}||\vec{w}|\cos(\theta) = 0 \; ?$$

Either $\vec{v}=\vec{0}$ or $\vec{w}=\vec{0}$ or $\underbrace{\cos(\theta)=0}_{\theta=\frac{\pi}{2}\ (90^\circ)}$.

Def: Two $\underbrace{\text{nonzero}}_{\vec{v}\neq\vec{0} \text{ and } \vec{w}\neq\vec{0}}$ vectors $\vec{v}$ and $\vec{w}$ are called perpendicular or orthogonal if the angle between them is $\theta = \frac{\pi}{2}$ (ie 90°)

Theorem: Two nonzero vectors $\vec{v}$ and $\vec{w}$ are perpendicular if and only if $\vec{v}\cdot\vec{w}=0$

Ex:

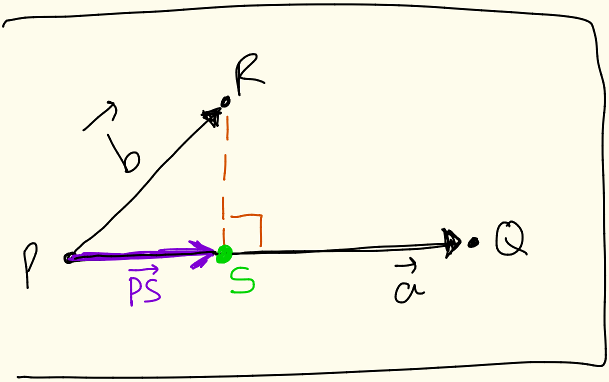


$$\vec{v} \cdot \vec{w} = (1)(2) + (-2)(1) + (0)(0) = 0$$

So, $\vec{v}$ and $\vec{w}$ are perpendicular.

That is $\theta = \frac{\pi}{2}$ (90°).

Projections



Let $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ be representations of two vectors $\vec{a}$ and $\vec{b}$ respectively, ie the vectors have the same initial point

If S is the foot of the projection from R to the line containing $\overrightarrow{PQ}$, then the vector with representation $\overrightarrow{PS}$ is called the vector projection of $\vec{b}$ onto $\vec{a}$ and is denoted by $\text{proj}_{\vec{a}}(\vec{b})$