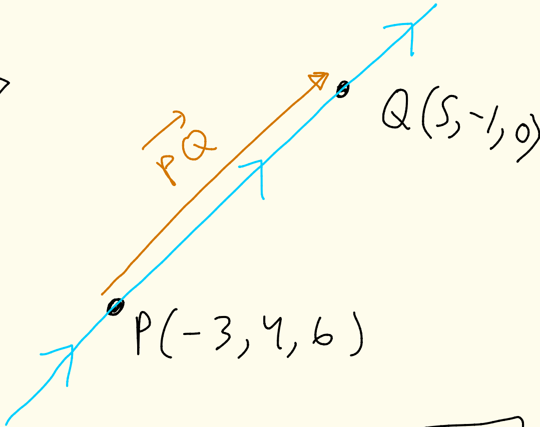
Math 2120

5/5/20

11.5

(15) Find an equation of the line through $P(-3,4,6)$ and $Q(5,-1,0)$

$$\vec{PQ} = \langle 5-(-3), -1-4, 0-6 \rangle$$
$$= \langle 8, -5, -6 \rangle$$



point on line
$P(-3,4,6)$

vector in direction of line / parallel to line
$\vec{PQ} = \langle 8, -5, -6 \rangle$

equation of line

$$x = -3 + 8t$$
$$y = 4 - 5t$$
$$z = 6 - 6t$$

t any real number

can use Q instead

another eqn

$$x = 5 + 8t$$
$$y = -1 - 5t$$
$$z = 0 - 6t$$

t any real #

20 Find an equation for the line through $(-3,4,2)$ that is perpendicular to both $\vec{u}=\langle 1,1,-5\rangle$ and $\vec{v}=\langle 0,4,0\rangle$

Given point on the line: $(-3,4,2)$

The cross product $\vec{u}\times\vec{v}$ will give us a vector that is perpendicular to both $\vec{u}$ and $\vec{v}$.

$$\vec{u}\times\vec{v}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\1&1&-5\\0&4&0\end{vmatrix}=\vec{i}\begin{vmatrix}1&-5\\4&0\end{vmatrix}-\vec{j}\begin{vmatrix}1&-5\\0&0\end{vmatrix}+\vec{k}\begin{vmatrix}1&1\\0&4\end{vmatrix}$$

$$=\vec{i}\left(0-(-20)\right)-\vec{j}(0-0)+\vec{k}(4-0)$$

$$=20\vec{i}+0\vec{j}+4\vec{k}=\langle 20,0,4\rangle$$

answer

$x=-3+20t$

$y=4+0t=4$

$z=2+4t$

t any real #

$\vec{v}\times\vec{u}=-\vec{u}\times\vec{v}$

$=\langle -20,0,-4\rangle$

This is ok too

11.6 pg 3

29 Find the unit tangent vector for the curve

$$\vec{r}(t) = \langle 6t, 6, \frac{3}{t} \rangle$$

in the range $0 < t < 2$
and specifically for $t = 1$.

$$\vec{r}(t) = \langle 6t, 6, 3t^{-1} \rangle$$

$$\vec{r}\,'(t) = \langle 6, 0, -3t^{-2} \rangle \leftarrow \boxed{\text{tangent vector}}$$

$$|\vec{r}\,'(t)| = \sqrt{6^2 + 0^2 + (-3t^{-2})^2} = \sqrt{36 + \frac{9}{t^4}}$$

unit tangent vector

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \frac{1}{\sqrt{36 + \frac{9}{t^4}}} \langle 6, 0, -\frac{3}{t^2} \rangle$$

$$= \left\langle \frac{6}{\sqrt{36 + 9t^{-4}}}, 0, \frac{-3t^{-2}}{\sqrt{36 + 9t^{-4}}} \right\rangle$$

$$\vec{T}(1) = \left\langle \frac{6}{\sqrt{45}}, 0, \frac{-3}{\sqrt{45}} \right\rangle$$

11.8

10 Find the arclength of the curve given by

$$\vec{r}(t) = \langle 3t-1,\ 4t+5,\ t \rangle$$

for $0 \leq t \leq 1$.

$$\vec{r}'(t) = \langle 3, 4, 1 \rangle$$

$$\text{arclength} = \int_0^1 |r'(t)|\, dt$$

$$= \int_0^1 \sqrt{3^2 + 4^2 + 1^2}\, dt$$

$$= \sqrt{26} \int_0^1 dt = \sqrt{26}\, t \Big|_0^1$$

$$= \sqrt{26}$$

Weds / Thurs / rest of today

I'll show you some more techniques not on the final

$$\int \tan^m(x)\sec^n(x)\,dx$$

when n is even, $n \geq 2$

- take out a $\sec^2(x)$ and save it
- Convert the remaining $\sec^2(x)$'s into tangent using $1+\tan^2(x) = \sec^2(x)$
- set $u = \tan(x)$, $du = \sec^2(x)\,dx$

Ex: $\int \tan^2(x)\sec^4(x)\,dx$

$$= \int \tan^2(x)\sec^2(x)\underbrace{\sec^2(x)\,dx}_{du\ (\text{later})}$$

$$= \int \tan^2(x) \sec^2(x) \sec^2(x)\, dx$$

$$= \int \tan^2(x) \left[1 + \tan^2(x)\right] \sec^2(x)\, dx$$

$$= \int u^2 (1 + u^2)\, du$$

$u = \tan(x)$

$du = \sec^2(x)\, dx$

$$= \int (u^2 + u^4)\, du$$

$$= \frac{u^3}{3} + \frac{u^5}{5} + C$$

$$= \boxed{\frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5} + C}$$