Math 2120

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$\boxed{10.2}$

34 Convert the polar equation to Cartesian coordinates (xy).

$$r = \frac{\sin(\theta)}{\sec^2(\theta)}$$

you can have $r=0$ here and $\theta=\pi$. That gives $0=0$.

$$\boxed{\begin{aligned} x &= r\cos(\theta) \\ y &= r\sin(\theta) \\ r^2 &= x^2+y^2 \end{aligned}}$$

So the origin is a solution to

$$\boxed{r = \sin(\theta)\cos^2(\theta)}$$

$r \neq 0$

$$r = \frac{y}{r}\cdot\frac{x^2}{r^2}$$

$$r^4 = yx^2$$

$$\underbrace{(x^2+y^2)^2}_{(r^2)^2} = yx^2$$

$$\boxed{x^4+2x^2y^2+y^4 = yx^2}$$

$(x,y)=(0,0)$ is a sol. to this curve so we didn't lose that solution when we divided by r pg 2

$$r = \sin(\theta)\sec^2(\theta) = \frac{\sin(\theta)}{\cos^2(\theta)}$$

$$\cos^2(\theta)\, r = \sin(\theta)$$

$$\frac{x^2}{r^2}\, r = \frac{y}{r}$$

$$\boxed{x^2 = y}$$

$x = r\cos(\theta)$
$y = r\sin(\theta)$
$r^2 = x^2 + y^2$

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$$\int \frac{dx}{(1-x^2)^{3/2}} = \int \frac{dx}{\left(\sqrt{1-x^2}\right)^3}$$

$$= \int \frac{\cos(\theta)}{\left(\sqrt{1-\sin^2(\theta)}\right)^3}\, d\theta =$$

$\sqrt{a^2 - x^2}$
$x = a\sin(\theta)$
$-\pi/2 \le \theta \le \pi/2$

$x = \sin(\theta)$
$dx = \cos(\theta)\, d\theta$

$\sqrt{\cos^2(\theta)}$
$= |\cos(\theta)|$
$= \cos(\theta)$

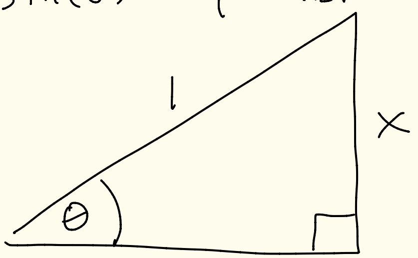
$$\int \frac{\cos(\theta)}{\cos^3(\theta)}\,d\theta = \int \frac{d\theta}{\cos^2(\theta)}$$

$$= \int \sec^2(\theta)\,d\theta = \tan(\theta) + C$$

$$= \boxed{\frac{x}{\sqrt{1-x^2}} + C}$$

$x = \sin(\theta)$

$\sin(\theta) = \frac{x}{1} = \frac{\text{opp}}{\text{hyp}}$



$? = \sqrt{1-x^2}$

$?^2 + x^2 = 1^2 \Rightarrow ? = \sqrt{1-x^2}$

$\tan(\theta) = \frac{\text{opp}}{\text{adj}}$

$= \frac{x}{\sqrt{1-x^2}}$

$\boxed{9.4}$

55 $$\sum_{k=0}^{\infty} \frac{x^k}{2^k} = \sum_{k=0}^{\infty} \left(\frac{x}{2}\right)^k$$

$$= \frac{1}{1 - \frac{x}{2}} = \frac{2}{2-x}.$$

$$\boxed{\begin{gathered} \left|\frac{x}{2}\right| < 1 \\ |x| < 2 \\ -2 < x < 2 \end{gathered}}$$

Not on final

If you're doing partial fractions and the denominator has $(x-a)^r$ where $r>1$ then for that term you use the fractions

$$\frac{A_1}{(x-a)} + \frac{A_2}{(x-a)^2} + \cdots + \frac{A_r}{(x-a)^r}$$

<u>Ex:</u> $\displaystyle\int \frac{dx}{(x-1)(x+1)^2}$

$$\frac{1}{\underbrace{(x-1)}\underbrace{(x+1)^2}} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

Multiply through by $(x-1)(x+1)^2$ to get:

$$1 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$

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$$\boxed{1 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)}$$

Find A, plug in $x=1$:

$$1 = A(2)^2 + B(0)(2) + C(0)$$

$$A = 1/4$$

Find C, plug in $x=-1$:

$$1 = A(0)^2 + B(-2)(0) + C(-2)$$

$$C = \frac{-1}{2}$$

To find B, plug in $x=0$, $A = 1/4$, $C = -1/2$:

$$1 = \frac{1}{4}(0+1)^2 + B(0-1)(0+1) - \frac{1}{2}(0-1)$$

$$1 = \frac{1}{4} - B + \frac{1}{2}$$

$$B = \frac{1}{4} + \frac{1}{2} - 1 = -1/4$$

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$$\int \frac{dx}{(x-1)(x+1)^2} = \int \left[\frac{1/4}{x-1} + \frac{-1/4}{x+1} + \frac{-1/2}{(x+1)^2} \right] dx$$

$$= \frac{1}{4} \ln|x-1| - \frac{1}{4} \ln|x+1| - \frac{1}{2}\left(-\frac{1}{x+1}\right) + C$$

$$\boxed{\int (x+1)^{-2}\, dx = -(x+1)^{-1} + C}$$

$$= \boxed{\frac{1}{4} \ln|x-1| - \frac{1}{4} \ln|x+1| + \frac{1/2}{x+1} + C}$$