Math 2120

5/7/20

Last day!

8.5

(pg 1)

29 Converge or diverge?

$$\sum_{k=1}^{\infty} \frac{k^2-1}{k^3+4} = 0 + \frac{3}{12} + \frac{8}{31} + \cdots$$

Spider sense
When k is large, $\frac{k^2-1}{k^3+4} \approx \frac{k^2}{k^3} = \frac{1}{k}$

Since $\sum \frac{k^2-1}{k^3+4}$ and $\sum \frac{1}{k}$ are series with positive terms, we can use the limit comparison test.

$$L = \lim_{k\to\infty} \frac{\left(\frac{k^2-1}{k^3+4}\right)}{\left(\frac{1}{k}\right)} = \lim_{k\to\infty} \left(\frac{k}{1}\right)\left(\frac{k^2-1}{k^3+4}\right)$$

$$= \lim_{k\to\infty} \frac{k^3-k}{k^3+4} = \lim_{k\to\infty} \frac{1-1/k^2}{1+4/k^3} = \frac{1-0}{1+0} = 1$$

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Since $0 < L < \infty$, either both series converge or both series diverge.

Since $\sum_{k=1}^{\infty} \frac{1}{k}$ diverges, so does $\sum_{k=1}^{\infty} \frac{k^2-1}{k^3+4}$

11.3

(31) Calculate $\text{proj}_{\vec{v}}(\vec{u})$ and $\text{scal}_{\vec{v}}(\vec{u})$

$\vec{u} = \langle 3,3,-3 \rangle$, $\vec{v} = \langle 1,-1,2 \rangle$

$$\text{proj}_{\vec{v}}(\vec{u}) = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\vec{v} = \underbrace{\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}}_{\text{scal}_{\vec{v}}(\vec{u})} \cdot \frac{\vec{v}}{|\vec{v}|}$$

$$\vec{u}\cdot\vec{v} = (3)(1)+(3)(-1)+(-3)(-2) = 6$$

$$|\vec{v}| = \sqrt{1^2+(-1)^2+2^2} = \sqrt{6}$$

$$\text{scal}_{\vec{v}}(\vec{u}) = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|} = \frac{6}{\sqrt{6}}$$

$$\text{proj}_{\vec{v}}(\vec{u}) = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\vec{v} = \frac{6}{(\sqrt{6})^2}\langle 1,-1,2\rangle = \langle 1,-1,2\rangle = \vec{v}$$

$$\boxed{7.8 \# 41}$$

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(41) $\displaystyle\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \lim_{t\to 0^+} \int_t^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}$

$\displaystyle = \lim_{t\to 0^+} 2e^{\sqrt{x}}\Big|_t^1 = \lim_{t\to 0^+}\left[2e^{\sqrt{1}} - 2e^{\sqrt{t}}\right]$

$= 2e - 2e^0$

$= \boxed{2e-2}$

$\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int 2e^u\,du$

$u = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$2\,du = \frac{dx}{\sqrt{x}}$

$= 2e^u + C$

$= 2e^{\sqrt{x}} + C$

So $\displaystyle\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ converges to $2e-2$.

9.2 Interval of convergence?

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18 $$\sum_{k=0}^{\infty} (-1)^k \frac{x^k}{5^k}$$

Could use ratio test, but in this case, its the geometric series.

$$\sum_{k=0}^{\infty} (-1)^k \frac{x^k}{5^k} = \sum_{k=0}^{\infty} \left(-\frac{x}{5}\right)^k = \frac{1}{1-\left(-\frac{x}{5}\right)}$$

This series converges precisely when

$\left|-\frac{x}{5}\right| < 1$ or $\frac{|-x|}{|5|} < 1$ or $|x| < 5$

or $-5 < x < 5$

$\boxed{\text{Not on test}}$

$$\int \tan^m(x)\sec^n(x)\,dx$$ where m is odd $m \geq 1$, $n \geq 1$.

- take out one $\sec(x)\tan(x)$ and save it for du
- rewrite the remaining $\tan^2(x)$'s in terms of $\sec(x)$ using $1+\tan^2(x) = \sec^2(x)$
- Set $u = \sec(x)$, $du = \sec(x)\tan(x)\,dx$

Ex: $\int \tan^5(x)\sec^3(x)\,dx$

$$= \int \underbrace{\tan^4(x)}_{\text{turn into } \sec(x)\text{'s}} \sec^2(x) \underbrace{\sec(x)\tan(x)\,dx}_{\text{save for } du}$$

$$= \int (\tan^2(x))^2 \sec^2(x) \sec(x)\tan(x)\,dx$$

$$= \int (\sec^2(x)-1)^2 \sec^2(x)\sec(x)\tan(x)\,dx$$

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$$= \int \left(\sec^2(x)-1\right)^2 \sec^2(x)\sec(x)\tan(x)\,dx$$

$$= \int (u^2-1)^2 u^2\,du = \int (u^4-2u^2+1)u^2\,du$$

$u = \sec(x)$
$du = \sec(x)\tan(x)\,dx$

$$= \int (u^6-2u^4+u^2)\,du$$

$$= \frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} + C$$

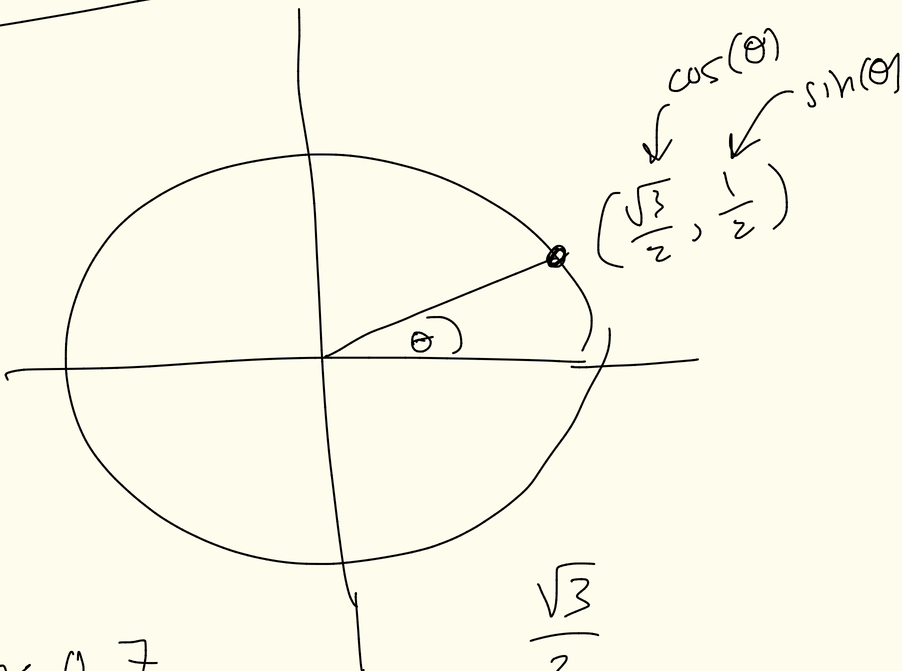
$$= \boxed{\frac{\sec^7(x)}{7} - \frac{2\sec^5(x)}{5} + \frac{\sec^3(x)}{3} + C}$$

Ex of what you can put on sheet for final

$\sum a_k, \sum b_k$ positive terms

$L = \lim_{k \to \infty} \frac{a_k}{b_k}$ if $0 < L < \infty$ then both converge or diverge

I'll give you the unit circle



$\frac{\sqrt{3}}{2}$

$\frac{\sqrt{2}}{2} \approx 0.7$