Math 4460 - Test 1 - Spring 2025

Name:____

Testing rules:

You may use a calculator that is not your phone. It cannot be internet-enabled.

Score			
1		2	
3		4	
5			
Total			

1. [10 points] Use the Euclidean algorithm to find $\gcd(356, 104)$.

You must use the Euclidean algorithm to get credit.

2. [10 points] Determine if there exist integers x and y such that

$$100x + 26y = 2$$

If there are solutions: first find a particular solution, and then find a formula for all solutions. If there are no solutions, then say why there are no solutions.

The following may be helpful for you:

$$\begin{array}{ccccc} 100 & = & 3 \cdot 26 & + & 22 \\ 26 & = & 1 \cdot 22 & + & 4 \\ 22 & = & 5 \cdot 4 & + & 2 \\ 4 & = & 2 \cdot 2 & + & 0 \end{array}$$

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3. [20 points - 10 each]

(a) Do there exist integers x and y such that $12x + 8y = 14$? If so, find an example of such an x and y. If not, explain why not.

(b) Let a, b be integers where $a \neq 0$. Prove that if $a|b$, then $a^2|b^2$.

4. [10 points] PICK ONE of the following proofs.

Do not do both. If you do both, then I will grade A.

A) Let x, y, z be integers with $x \neq 0$. Prove that if $x|yz$, then $\left.\dfrac{x}{\gcd(x,y)}\,\right|\, z$.

B) Prove that 4 does not divide $n^2 + 2$ for any integer n.

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5. [10 points] PICK ONE of the following proofs.

Do not do both. If you do both then I will grade C.

C. Let a, b be positive integers. Let $x = \gcd(a, b)$ and $y = \gcd(a, a + b)$. Prove that $x \leq y$.

D) Let a, b, c be postive integers. Prove that if $\gcd(a, b) = 1$ and $c|a$, then $\gcd(b, c) = 1$.
