Math 4740 - Fall 2024 - Test 1

Name:

Score	
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Total	

1. [12 points - 6 each]

(a) Suppose that a license plate consists of two letters (taken from $A - Z$), followed by four numbers (taken from $0 - 9$), followed by two symbols (where the symbols are either a circle or a star). The only restriction is that the first letter on the plate cannot be a Z. How many different license plates are there? Note that there are 26 letters in the alphabet.

Two example license plates are $AZ0012 \star \square$ and $BB1234\square\square$.

$ZQ1744 \star \star$ is not a license plate because of the first letter being a Z

(b) Consider a permutation of the six digits $1, 2, 3, 4, 5, 6$. How many such permutations are there where the 5 must occur in the second spot of the permutation? Some examples are 251346 and 152346 and 654321.

2. [12 points - 4 each] Suppose you want to model the experiment of rolling two 3-sided dice. Here each die has sides $1, 2, 3$ and each side is equally likely.

(a) Write down the elements of a sample space S for such an experiment. List all of the elements of S.

(b) Let E be the event where the first die is a 1. Let F be the event where the second die is a 2. List the elements of E and F and find $E \cap F$ and $E \cup F$ and $\overline{F}$.

(c) Calculate the probability of E and the probability of F.

3. [12 points - 6 each]

   (a) Suppose that four 8-sided dice are thrown. What is the probability that you get exactly two 3's?

   (b) Suppose a coin is tossed 20 times. What is the probability that at least 2 heads occurs?

Recall that a standard 52-card deck has 4 suits: $\clubsuit$, $\heartsuit$, $\spadesuit$, $\diamondsuit$.
For each suit we have the following 13 face values: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K.

4. [12 points - 6 each] Suppose a dealer deals you 4 cards from a standard 52-card deck.

(a) What is the size of the sample space for this experiment?

(b) What is the probability that you all of the cards that you get have the same suit?
Two examples are:

Example 1: $6\spadesuit$, $8\spadesuit$, $K\spadesuit$, $A\spadesuit$

Example 2: $2\diamondsuit$, $5\diamondsuit$, $J\diamondsuit$, $9\diamondsuit$

5. [12 points - 6 each] Consider a box with 12 balls labeled $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$.
Suppose you randomly draw three balls from the box. You grab them at the same time, so
order doesn't matter. For example, maybe you pick balls $2, 5, 9$ which is the same as $5, 9, 2$.

(a) What is the sample size of this experiment?

(b) What is the probability that all three balls are even numbered?
(For example, $2, 6, 8$ are chosen.)

(c) What is the probability that one ball is even and the other two balls are odd?
(For example, $2, 7, 9$ are chosen.)