

California State University, Los Angeles

Syllabus for the Analysis Comprehensive Exam

This examination involves topics mainly from MATH 4650, and MATH 5021.

Specifically, students are expected to know the following basics from MATH 4650 (only \mathbb{R}):

- (1) Sequences of real numbers, limits, convergence (using epsilon- N proofs)
- (2) Series of real numbers, limit as a sequential limit of partial sums, tests for convergence of series
- (3) Bounded sequences, Cauchy sequences, Cauchy convergence criterion
- (4) Cluster points, liminf and limsup of a sequence
- (5) Topology: open, closed, connected sets, including definitions and examples (closure, boundary, interior of a set)
- (6) Compact sets, including definitions and examples
- (7) Bolzano-Weierstrass Theorem
- (8) Heine-Borel Theorem
- (9) Limits and continuity of functions from \mathbb{R} to \mathbb{R}

References:

Elementary Classical Analysis, J. Marsden and M. Hoffman
Introduction to Real Analysis, R. Bartle and D. Sherbert
Elements of Real Analysis, H. Gaskill and P. Narayanaswami
Introduction to Real Analysis, Vol 1, J. Lebl

Also, students are expected to know the following basics from MATH 5021:

- (1) Vector spaces (finite and infinite dimensional)
- (2) Metrics, norms, and inner products (in finite and infinite dimensional spaces)
- (3) Bases, orthonormal bases, orthogonalization
- (4) Projections and approximation
- (5) Convergence of sequences and series of vectors
- (6) Hilbert spaces, Bessel and Parseval relations
- (7) Fourier series and generalized Fourier series. Bessel and Parseval relations
- (8) Contraction mapping theorem
- (9) Boundedness, continuity, and the norm of a linear operator

References:

Introduction to Functional Analysis with Applications, E. Kreyszig
An Introduction to Hilbert Space, N. Young
Methods of Mathematical Physics, Vol 1, R. Courant and D. Hilbert