In[58]:=
```
(*********************)
(* Helper functions *)
(*********************)

(* returns a list of all the
 Gaussian Integers with norm equal to n *)
normSolver[n_] := Module[{a, b, sqrtn, answerlist},
   sqrtn = Sqrt[n];
   answerlist = {};
   For[a = Floor[-sqrtn], a ≤ sqrtn,  a++,
      For[b = Floor[-sqrtn], b ≤ sqrtn, b++,
         If[Equal[n, a * a + b * b],
            answerlist = Append[answerlist, a + b * I];
        ];
        ];
   ];

   Return[answerlist];
  ];

(* If z and w are Gaussian integers,
this module returns True if z divides w, otherwise
 it returns False *)
gaussianDivides[z_, w_] := Module[{quotient},
   quotient = Simplify[w / z];
   If[IntegerQ[Re[quotient]] && IntegerQ[Im[quotient]],
    Return[True];
   ];
   Return[False];
  ];
```

In[69]:=
```
(****************)
(* Main program *)
(****************)

bound = 20;
divisorList = {};
For[n = 1, n ≤ bound, n++,
  (* Try to find z and w where N(z)=d divides N(w)=n,
  but z does NOT divide w. *)
  divisorListForN = Divisors[n];
  possibleWs = normSolver[n];

  (* TESTING
   Print["n = ", n];
  Print["divisorList = ", divisorList];
  Print["possibleWs = ", possibleWs];*)

  (* Go through the possible w's,
  then the possible d's, then the possible z's *)
  For[i = 1, i ≤ Length[possibleWs], i++,
     w = possibleWs[[i]];

     For[j = 1, j ≤ Length[divisorListForN], j++,
       d = divisorListForN[[j]];
       possibleZs = normSolver[d];

       For[k = 1, k ≤ Length[possibleZs], k++,
           z = possibleZs[[k]];

           If[(! Equal[d, 1]) &&
       (! Equal[d, n]) && (! gaussianDivides[z, w]),
```

```
            Print["z = ", z , " and ",
        "w = ", w , " satisfy N(z) = ",
                    d, " and N(w) = ", n,

        " but, z does not divide w since w/z = ",
        Simplify[w / z] ];
           ];
        ];
      ];
  ];
 ];
```

z = −2 − 𝕚 and w = −3 − 𝕚 satisfy N(z) = 5 and N(w) = 10 but, z does not divide w since w/z = $\frac{7}{5} - \frac{𝕚}{5}$

z = −1 + 2 𝕚 and w = −3 − 𝕚 satisfy N(z) = 5 and N(w) = 10 but, z does not divide w since w/z = $\frac{1}{5} + \frac{7\,𝕚}{5}$

z = 1 − 2 𝕚 and w = −3 − 𝕚 satisfy N(z) = 5 and N(w) = 10 but, z does not divide w since w/z = $-\frac{1}{5} - \frac{7\,𝕚}{5}$

z = 2 + 𝕚 and w = −3 − 𝕚 satisfy N(z) = 5 and N(w) = 10 but, z does not divide w since w/z = $-\frac{7}{5} + \frac{𝕚}{5}$

z = −2 + 𝕚 and w = −3 + 𝕚 satisfy N(z) = 5 and N(w) = 10 but, z does not divide w since w/z = $\frac{7}{5} + \frac{𝕚}{5}$

z = −1 − 2 𝕚 and w = −3 + 𝕚 satisfy N(z) = 5 and N(w) = 10 but, z does not divide w since w/z = $\frac{1}{5} - \frac{7\,𝕚}{5}$

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$z = 2 - i$ and $w = -3 + i$ satisfy $N(z) = 5$ and $N(w) = 10$ but, z does not divide w since $w/z = -\frac{7}{5} - \frac{i}{5}$

$z = -2 + i$ and $w = -1 - 3\,i$ satisfy $N(z) = 5$ and $N(w) = 10$ but, z does not divide w since $w/z = -\frac{1}{5} + \frac{7\,i}{5}$

$z = -1 - 2\,i$ and $w = -1 - 3\,i$ satisfy $N(z) = 5$ and $N(w) = 10$ but, z does not divide w since $w/z = \frac{7}{5} + \frac{i}{5}$

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$z = -2 + i$ and $w = 3 - i$ satisfy $N(z) = 5$ and $N(w) = 10$ but, z does not divide w since $w/z = -\frac{7}{5} - \frac{i}{5}$

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