Reference: Mechanics of Materials, Beer and Johnston, 3rd Ed., 2002, p. 240

Reinforced concrete problem

4.55 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

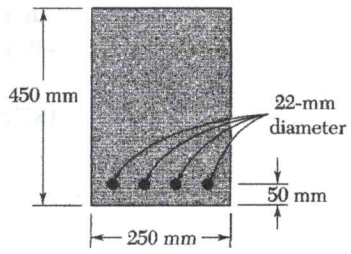
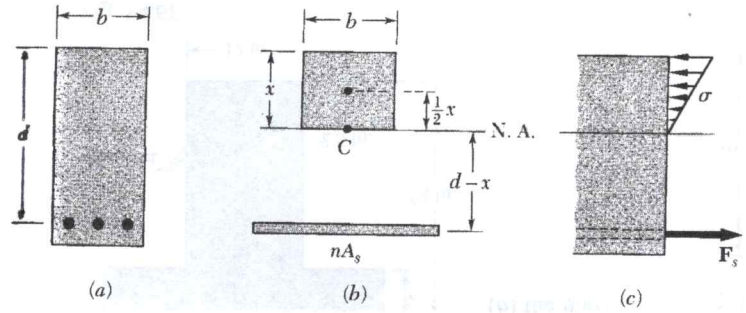


Fig. P4.55

Replace steel bars' total cross-sectional area with equivalent area: $A_s \cdot \frac{E_s}{E_c}$

Calculate location x of centroid C using the above area and neglecting the concrete area in tension..



$M := 175 \cdot kN \cdot m$

$E_c := 25 \cdot GPa$

$E_s := 200 \cdot GPa$

$b := 250 \cdot mm$

$d := 450 \cdot mm - 50 \cdot mm$ $\quad d = 400 \cdot mm$

$A_s := 4 \cdot \left[\pi \cdot \left(\frac{22}{2} \cdot mm \right)^2 \right]$ $\quad A_s = 1.521 \times 10^3 \cdot mm^2$

$n := \frac{E_s}{E_c}$ $\quad n = 8$

$n \cdot A_s = 1.216 \times 10^4 \cdot mm^2$

Determine location of C by calculating value of x from sum of first moments of areas about N. A.:

$$(b \cdot x) \cdot \frac{x}{2} - n \cdot As \cdot (d - x) := 0$$

$$\frac{b}{2} \cdot x^2 + n \cdot A_s \cdot x - n \cdot A_s \cdot d := 0$$

$$x := \frac{-\left(n \cdot A_s\right) + \sqrt{\left(n \cdot A_s\right)^2 + 2 \cdot b \cdot \left(n \cdot A_s \cdot d\right)}}{b}$$

$$x = 154.55 \cdot mm$$

$$I := \frac{1}{12} \cdot b \cdot x^3 + b \cdot x \cdot \left(\frac{x}{2}\right)^2 + n \cdot A_s \cdot (d - x)^2 \qquad I = 1.04 \times 10^{-3}\, m^4$$

Concrete $\sigma_c := \frac{M \cdot x}{I}$ $\sigma_c = 25.99 \cdot MPa$ Compression <---------- Ans.

Steel $\sigma_s := n \cdot \left[\frac{M \cdot (d - x)}{I}\right]$ $\sigma_s = 330 \cdot MPa$ Tension <---------- Ans.