Solutions to FE Exam "Dynamics" Review Problems

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Presented here are my solutions to the "Dynamics" review problems that used to be available online, until the end of 2017, on the website for the book: Beer and Johnston, *Vector Mechanics for Engineers, Statics and Dynamics*, Ninth Edition, 2010, at: http://highered.mcgraw-hill.com/sites/0073529400/information_center_view0/ . Before these review problems went "Out of Print," I downloaded and collected them in the following "problems.pdf" file: http://www.calstatela.edu/sites/default/files/users/u28426/felszeghy/problems.pdf .

My solutions, which you will find below, are for the review problems that are associated with the following chapters and topics in the above book:

Chpt. 11: Kinematics of Particles
Chpt. 12: Kinetics of Particles: Newton's Second Law
Chpt. 13: Kinetics of Particles: Energy and Momentum Methods
Chpt. 14: Systems of Particles
Chpt. 15: Kinematics of Rigid Bodies
Chpt. 16: Plane Motion of Rigid Bodies: Forces and Accelerations
Chpt. 17: Plane Motion of Rigid Bodies: Energy and Momentum Methods
Chpt. 19: Mechanical Vibrations

As I mentioned in class, some of the formerly available online problem statements had errors in them, and some online solutions and answers were wrong! For this reason, I have included in the above mentioned "problems.pdf" file a list of errors and corrections. Although I shared the errors and corrections with McGraw-Hill, the company never made any corrections to its Website.

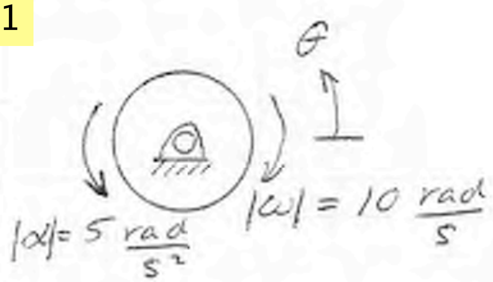
The formerly available online problems were not numbered; they were identified by chapter numbers only. For this reason, when I downloaded the online problems, I numbered them consecutively in a decimal format, XX.X, where XX refers to the chapter number, and X stands for the sequence number. All the downloaded problems numbered this way are included in the above mentioned "problems.pdf" file under the heading: "Part 1, FE Exam Review, Online Problems and Solutions."

My own solutions, which you will find below, follow the problem numbering scheme I established above. I included sketches in my solutions to allow you to identify more easily the problems to which my solutions apply.

I wish you all the best on your computer-based FE exam!

Chpt. 11

11.1



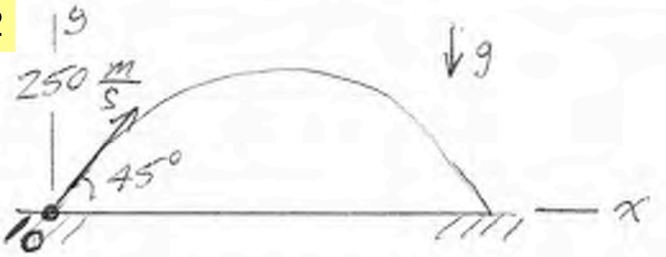
$\omega_0 = -10 \; \frac{rad}{s}$

$\alpha = 5 \; \frac{rad}{s^2}$

$\omega = \omega_0 + \alpha t = -10 + 5t$

$\omega = 0$ at $t = 2 s$ ← Ans.

11.2



$v_y = v_0 \sin\theta - gt$

$v_y = 0$ at $t_1 = \frac{v_0 \sin\theta}{g}$

$= \frac{250 \sin 45^\circ}{9.81}$

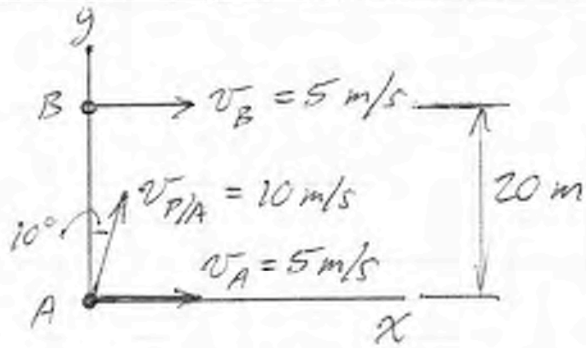
$= 18.02 s$

$y = 0$ at $t_2 = 36.04 s$

x @ $t_2 = 36.04 s$:

$x = v_0 (\cos\theta) t_2 = 250 (\cos 45^\circ) 36.04 = 6371 m$ ← Ans.

11.3



A and B remain fixed in moving x-y axes attached to A.

Puck motion in y-direction

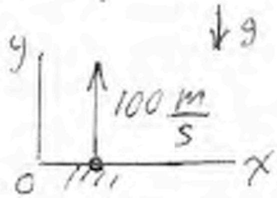
$y = v_{P/A} (\cos 10^\circ) t$

Puck reaches line of $\underline{v}_B$ at $t = \frac{20}{10 \cos 10^\circ} = 2.031 s$

x @ $t = 2.031 s$: $x = v_{P/A} (\sin 10^\circ) t = 10 (\sin 10^\circ) 2.031$

$= 3.53 m$ ← Ans.

11.4

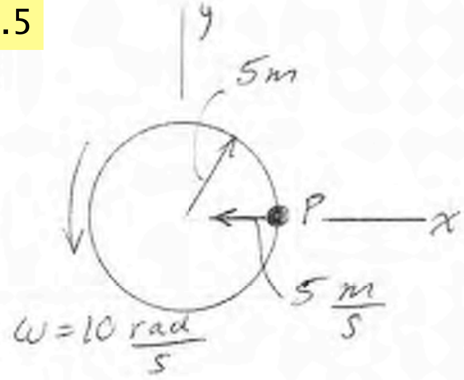


$v = v_0 + a_0 t \qquad a_0 = -g$

$v = 0$ at $t = \frac{v_0}{g} = \frac{100}{9.81} = 10.19 s$ ← Ans.

Note: $y = y_{max}$ when $v = 0$: $y_{max} = v_0 t - a_0 \frac{t^2}{2} = 509.7 m$

11.5



Use polar coordinates

$$a_r = \ddot{r} - r\dot{\theta}^2$$

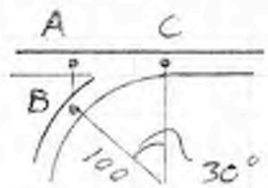
$$= 0 - (5)(10)^2 = -500 \ \frac{m}{s^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$= 0 + (2)(-5)(10) = -100 \ \frac{m}{s^2}$$

$$\underline{a} = -500\,\underline{e}_r - 100\,\underline{e}_\theta = -500\,\underline{i} - 100\,\underline{j}, \ \frac{m}{s^2} \quad \leftarrow \text{Ans.}$$

11.6



Time for B to arrive at C:

$$t = \frac{s_B}{v_B} = \frac{(\pi/6)100}{\left(\frac{90\times1000}{3600}\right)} = 2.094 \text{ s}$$

Time for A to arrive at C:

$$t = \frac{s_A}{v_A} = \frac{100 \sin 30°}{\left(\frac{100\times1000}{3600}\right)} = 1.800 \text{ s}$$

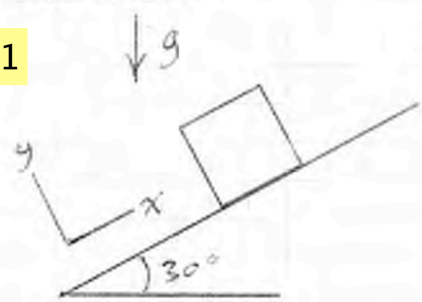
So A gets to C first and B gets to C by distance <u>B is behind A</u> when

$$d = (2.094 - 1.8)\left(\frac{100\times1000}{3600}\right)$$

$$= 8.17 \text{ m} \quad \leftarrow \text{Ans}$$

<u>Chpt. 12</u>

12.1



FBD

$$\Sigma F_x = ma \Rightarrow -mg \sin 30° = ma$$

$$a = -9.81 \sin 30° = -4.905 \ \frac{m}{s^2}$$

$$v = v_0 + at = 5 - 4.905\,t$$

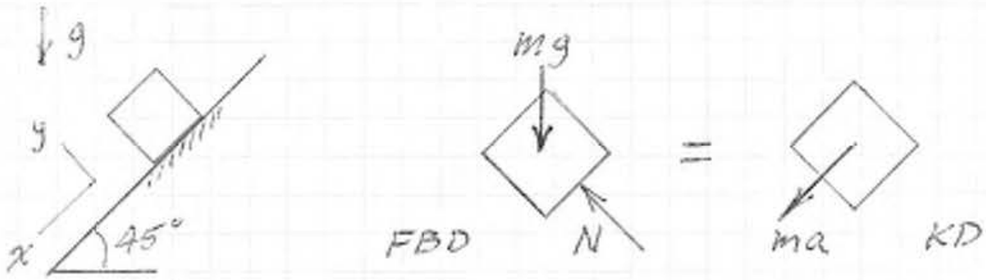
$$v = 0 \text{ when } t = \frac{5}{4.905} = 1.019 \text{ s}$$

$$x \text{ @ } t = 1.019\text{ s}: \quad x = v_0 t + \frac{at^2}{2} = (5)(1.019) - 4.905\frac{(1.019)^2}{2} = 2.55 \text{ m}$$

Ans.

INSERT AFTER 2

12.2



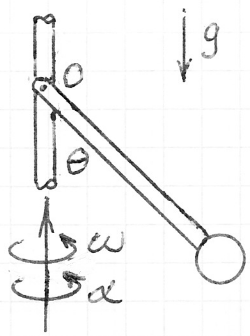
$$\Sigma F_x = ma$$

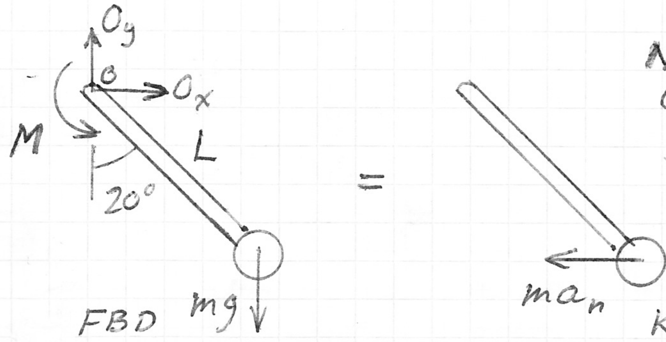
$$\cancel{m}g \sin 45° = \cancel{m}a$$

$$a = (9.81) \sin 45° = 6.94 \frac{m}{s^2}$$ ← Ans.

③

12.3





Note: Only <u>in-plane</u> forces and couple are shown.

$$\circlearrowleft \Sigma M_o = -ma_n L\cos 20°$$

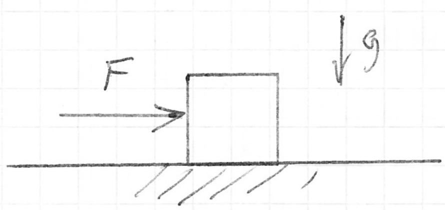
$$M - mgL\sin 20° = -m(\omega^2 L\sin 20°)L\cos 20°$$

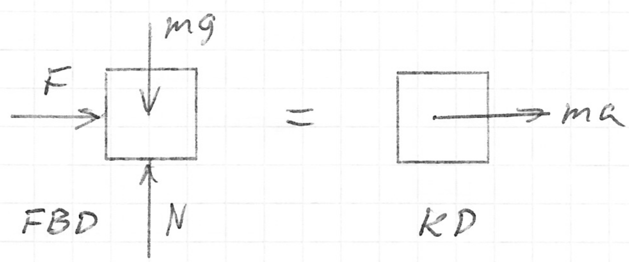
$$M = (5)(9.81)(2\sin 20°) - (5)\left(\frac{50\times 2\pi}{60}\right)^2(2\sin 20°)(2\cos 20°)$$

$$M = -142.7 \text{ N}\cdot\text{m}$$

$\underline{M} = 142.7$ N·m ↻ ← Ans.

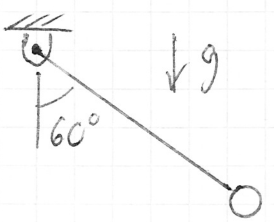
12.4

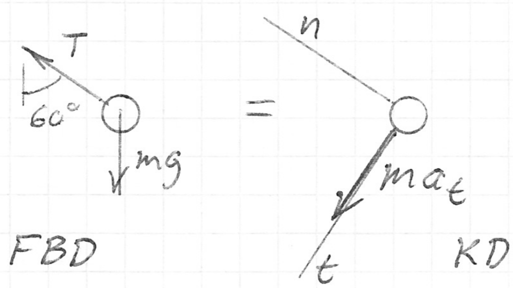




$F = ma = (4)(15) = 60$ N ← Ans.

12.5

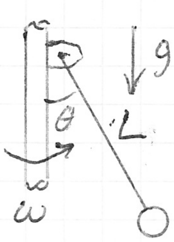


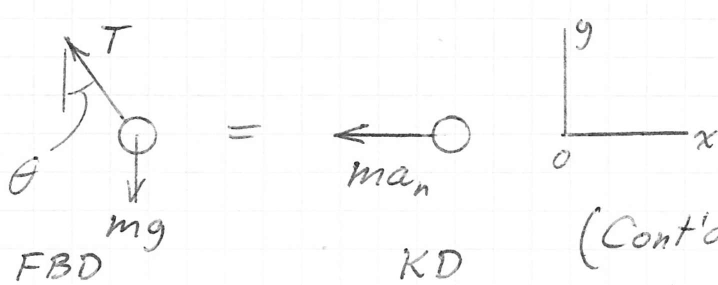


$\Sigma F_n = 0 \Rightarrow T - mg\cos 60° = 0$

$T = mg\cos 60° = (3)(9.81)\cos 60° = 14.72$ N ← Ans.

12.6





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$$\Sigma F_x = -ma_n \Rightarrow -T\sin\theta = -ma_n \quad (1)$$

$$\Sigma F_y = 0 \Rightarrow T\cos\theta - mg = 0 \quad (2)$$

Eliminate T between (1) & (2):

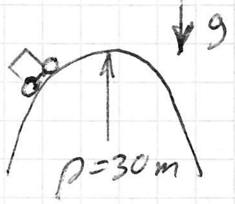
$$\tan\theta = \frac{\cancel{m}a_n}{\cancel{m}g}$$

$$= \frac{\omega^2 L \sin\theta}{g}$$

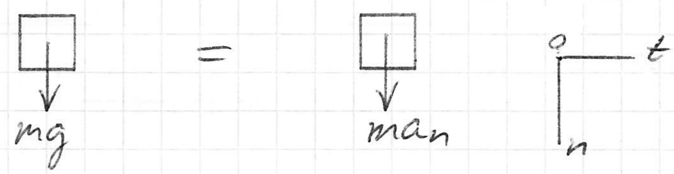
$$\cos\theta = \frac{g}{\omega^2 L} = \frac{9.81}{\left(\frac{20\times 2\pi}{60}\right)^2 4} = 0.559$$

$$\theta = 56.0^\circ \longleftarrow \text{Ans.}$$

12.7



FBD KD



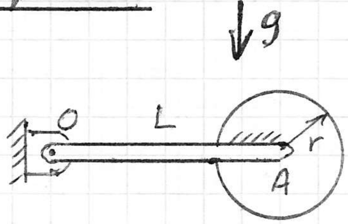
$$\Sigma F_n = ma_n$$

$$\cancel{m}g = \cancel{m}\frac{v^2}{\rho}$$

$$v^2 = \rho g = (30)(9.81)$$

$$v = 17.1 \text{ m/s} \longleftarrow \text{Ans.}$$

13.1 Chpt. 13



$$0 = \Delta T + \Delta V_g$$

$$\Delta T = \frac{1}{2} I_o \omega_2^2$$

$$= \frac{1}{2}\left[\frac{1}{3}m_r L^2 + \frac{1}{2}m_d r^2 + m_d L^2\right]\omega_2^2$$

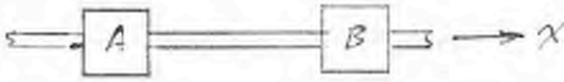
$$= \frac{1}{2}\left[\frac{1}{3}\frac{10}{9.81}1^2 + \frac{1}{2}10(0.3)^2 + 10(1)^2\right]\omega_2^2$$

$$= 5.39\,\omega_2^2$$

$$\Delta V_g = -m_r g\frac{L}{2} - m_d g L = -10(0.5) - 10(9.81)(1) = -103.1$$

$$\Delta T = -\Delta V_g \Rightarrow \omega_2^2 = 19.11, \quad \omega_2 = 4.37, \quad v_A = \omega_2 1 = 4.37\,\frac{m}{s} \longleftarrow \text{Ans.}$$

13.2



Conservation of total linear momentum:

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$(10)(10) + (20)(-15) = 10 v_A' + 20 v_B'$$

$$-200 = 10 v_A' + 20 v_B' \quad (1)$$

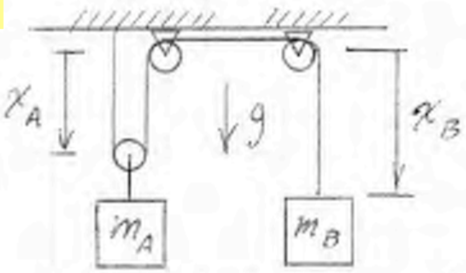
$$e = \frac{v_B' - v_A'}{v_A - v_B} \Rightarrow 0.6 = \frac{v_B' - v_A'}{10 - (-15)} \Rightarrow 15 = -v_A' + v_B' \quad (2)$$

$$(1) - 20 \times (2) \Rightarrow -200 - 300 = 10 v_A' + 20 v_A'$$

$$30 v_A' = -500$$

$$v_A' = -16.67 \ \frac{m}{s} \quad \leftarrow \text{Ans.}$$

13.3

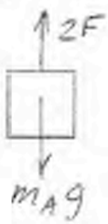


Cable length is const. Therefore,
$2x_A + x_B + \text{const.} = \text{const.}$

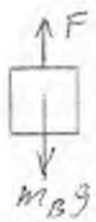
$$2\Delta x_A = -\Delta x_B \quad (1)$$

$$2\dot{x}_A = -\dot{x}_B \quad (2)$$

FBD



FBD



Work - energy:

$$m_A g \Delta x_A - 2F\Delta x_A = \frac{1}{2} m_A \dot{x}_A^2 \quad (3)$$

$$m_B g \Delta x_B - F\Delta x_B = \frac{1}{2} m_B \dot{x}_B^2 \quad (4)$$

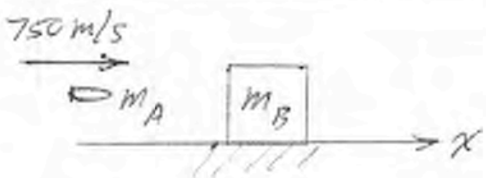
Substitute (1) into (3), and (2) into (4), and add resulting equations

$$(-m_A g/2 + m_B g)\Delta x_B = \frac{1}{2}(m_A \dot{x}_A^2 + m_B 4\dot{x}_A^2)$$

$$9.81(-15 + 40)(10) = (15 + 80)\dot{x}_A^2$$

$$\dot{x}_A^2 = 25.82, \quad \dot{x}_A = 5.08 \ \frac{m}{s} \quad \leftarrow \text{Ans.}$$

13.4



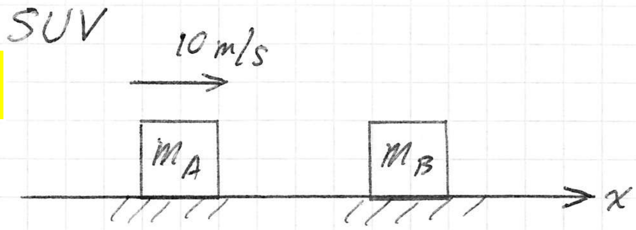
Conservation of total linear momentum:

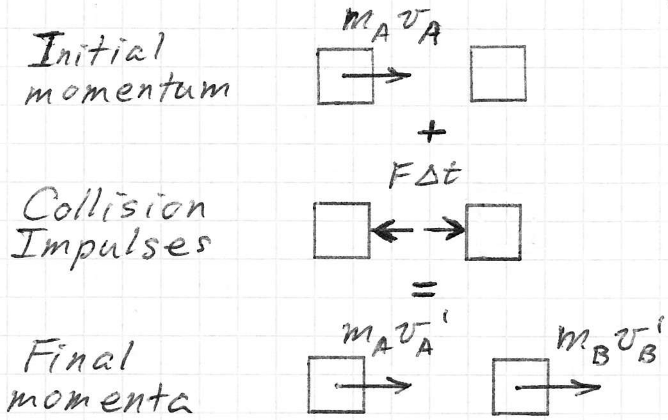
$$m_A v_A + \cancel{m_B v_B}^{\,0} = (m_A + m_B)v'$$

$$v' = \frac{m_A v_A}{m_A + m_B} = \frac{(0.015)(750)}{10.015}$$

$$v' = 1.123 \ \frac{m}{s} \quad \leftarrow \text{Ans}$$

13.5





$$e = \frac{v_B' - v_A'}{v_A - \cancel{v_B}^{\,0}}$$

$$0.7 = \frac{v_B' - v_A'}{10} \quad (1)$$

Conservation of total linear momentum:

$$m_A v_A + m_B \cancel{v_B}^{\,0} = m_A v_A' + m_B v_B'$$

$$(6000)(10) = 6000 v_A' + 4000 v_B'$$

or $$10 = v_A' + \frac{2}{3} v_B' \quad (2)$$

Add (1) & (2): $$17 = \frac{5}{3} v_B'$$

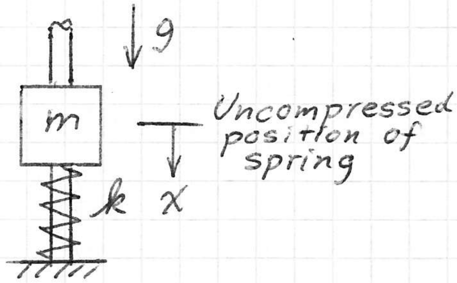
$$v_B' = 10.2 \text{ m/s}$$

Apply impulse-momentum eq. for B:

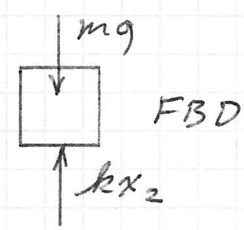
$$F\Delta t = m_B v_B'$$

$$F = \frac{m_B v_B'}{\Delta t} = \frac{(4000)(10.2)}{0.3} = 136 \text{ kN} \longleftarrow \text{Ans.}$$

13.6



$x_1 = 0.02$ m, $v_1 = 0$ m/s.
$v_2 = v_{max}$ when $a_2 = 0$.



$$kx_2 = mg$$

$$x_2 = \frac{(10)(9.81)}{10^5}$$

$$= 98.1 \times 10^{-5} \text{ m}$$

$k = 0.1$ kN/mm (NEW)

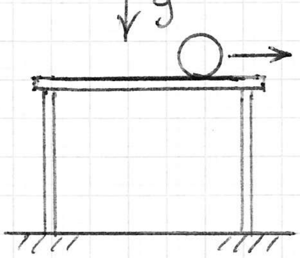
$$0 = \Delta T + \Delta V_g + \Delta V_e$$

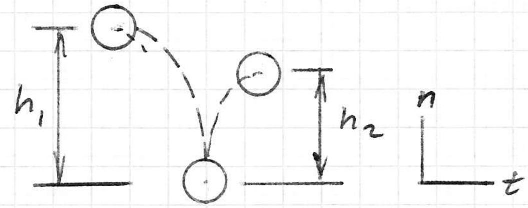
$$0 = \frac{1}{2} m v_{max}^2 - mg(x_2 - x_1) + \frac{1}{2} k (x_2^2 - x_1^2)$$

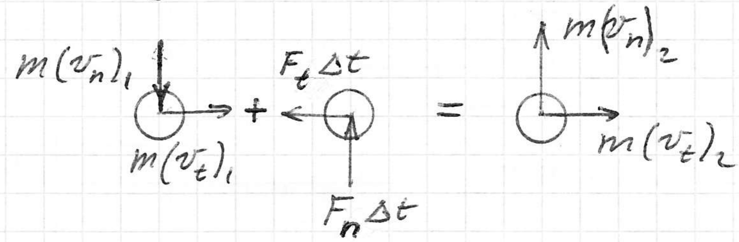
$$v_{max}^2 = \frac{2}{10}\left[-(10)(9.81)(0.02 - 98.1\times10^{-5}) + \frac{1}{2}10^5(0.02^2 - 98.1^2\times10^{-10})\right]$$

$$v_{max}^2 = 3.617 \Longrightarrow v_{max} = 1.902 \frac{\text{m}}{\text{s}} \longleftarrow \text{Ans.}$$

13.7







$$0 = \Delta T + \Delta V_g \Rightarrow \quad \tfrac{1}{2} m[(v_n)_1]^2 = mgh_1 \quad (1)$$

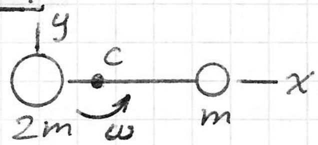
$$\tfrac{1}{2} m[(v_n)_2]^2 = mgh_2 \quad (2)$$

Solve (1) and (2) for $(v_n)_1$ and $(v_n)_2$, and substitute in eq. below:

$$e = \frac{(v_n)_2}{(v_n)_1} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{0.8}{1}} = 0.894 \quad \longleftarrow \text{Ans.}$$

14.1

Chpt. 14



Center of mass: $x_c = \dfrac{3m}{3m}$

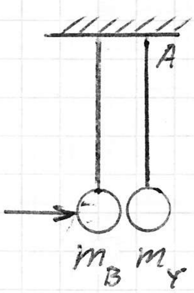
$= 1 \text{ m}$

$$H_c = 2 \times m(2\omega) + 1 \times 2m(1\omega)$$

$$= 6m\omega = 6(1)(5) = 30 \ \frac{\text{kg}\cdot\text{m}^2}{\text{s}} \quad \longleftarrow \text{Ans.}$$

14.2

See middle p. 8



Conserv. of total linear momentum:

$$m_B v_B + \cancel{m_Y v_Y}^{\,0} = m_B v_B' + m_Y v_Y'$$

$$(0.5)(6) = 0.5 v_B' + (1) v_Y' \quad (1)$$

$$e = \frac{v_Y' - v_B'}{v_B}$$

$$1 = \frac{v_Y' - v_B'}{6} \Rightarrow 6 = -v_B' + v_Y' \quad (2)$$

$$(1) + 0.5 \times (2) \Rightarrow \quad 6 = 1.5 v_Y' \Rightarrow v_Y' = 4 \ \frac{\text{m}}{\text{s}}$$

$$H_A = 3 \times m_Y v_Y' = 3 \times (1)(4) = 12 \ \frac{\text{kg}\cdot\text{m}^2}{\text{s}} \quad \longleftarrow \text{Ans.}$$

14.3

$m = 5\ kg \qquad \underline{r} = 10\underline{i} - 2\underline{j} + 5\underline{k},\ m$

$\underline{v} = 3\underline{i} + 2\underline{j} - 5\underline{k},\ m/s$

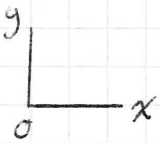
$$\underline{H}_o = \underline{r} \times m\underline{v} = 5 \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 10 & -2 & 5 \\ 3 & 2 & -5 \end{vmatrix}$$

$$= 5\left[\underline{i}(10-10) - \underline{j}(-50-15) + \underline{k}(20+6)\right]$$

$$= 325\,\underline{j} + 130\,\underline{k},\ \frac{kg \cdot m^2}{s} \quad \longleftarrow \text{Ans.}$$

14.4

See 7 bottom $\Longrightarrow$ $v_Y' = 4\ \frac{m}{s}$

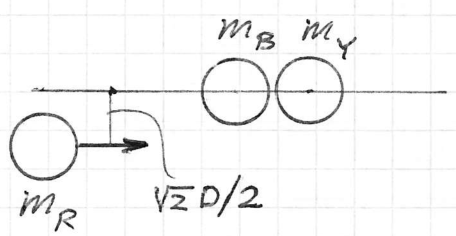


$m_Y:\ 0 = \Delta T + \Delta V_g$

$$0 = \left[0 - \tfrac{1}{2} m_Y (v_Y')^2\right] + m_Y g\left[y_2 - 0\right]$$

$$y_2 = \frac{\frac{1}{2}\cancel{m_Y}(4)^2}{\cancel{m_Y}\,g} = 0.815\ m \quad \longleftarrow \text{Ans.}$$

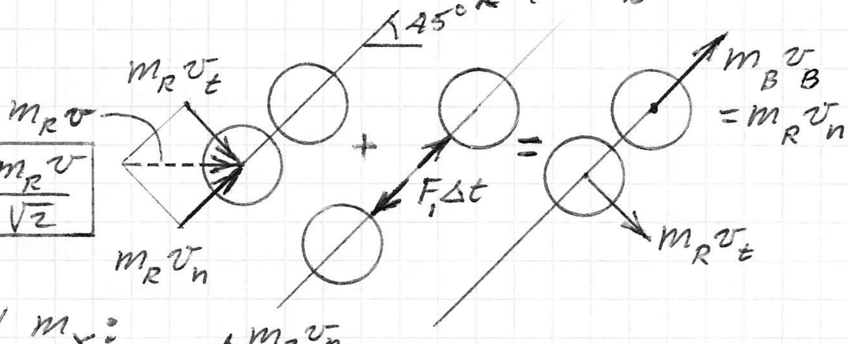
14.5



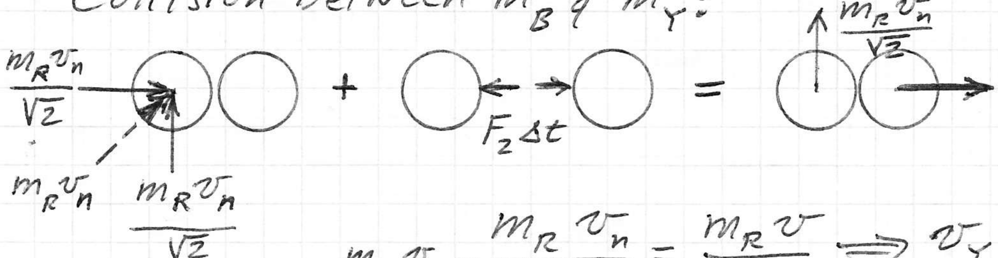
$v = 5\ \frac{m}{s}$

Collision between m_R & m_B:

$$\boxed{m_R v_n = \frac{m_R v}{\sqrt{2}}}$$



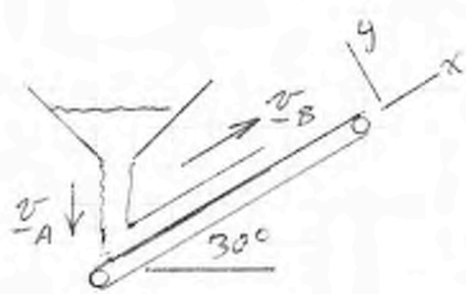
Collision between m_B & m_Y:



$m_Y v_Y = \frac{m_R v_n}{\sqrt{2}}$

$$m_Y v_Y = \frac{m_R v_n}{\sqrt{2}} = \frac{m_R v}{2} \Longrightarrow v_Y = \frac{v}{2} = 2.5\ \frac{m}{s} \quad \longleftarrow \text{Ans.}$$

14.6



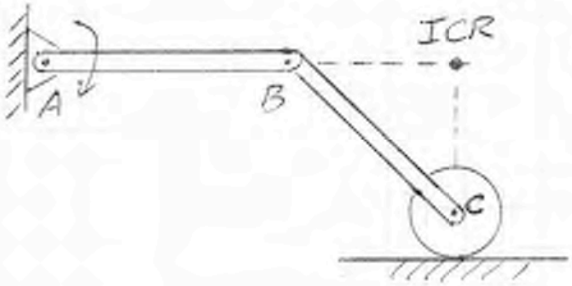
$$\Sigma \underline{F} = \frac{dm}{dt}\left(\underline{v}_B - \underline{v}_A\right)$$

$$\Sigma F_x = \frac{20000}{9.81 \times 3600}\left(0.5 - (-0.5 \sin 30°)\right)$$

$$= 0.425 \text{ N}$$ ← Ans.

Chpt. 15

15.1



$\underline{\omega}_{AB} = 10 \frac{rad}{s}$ ↻

$\underline{\alpha}_{AB} = 0 \frac{rad}{s^2}$

$$\underline{a}_c = \underline{a}_B + \underline{a}_{c/B} \quad (1)$$

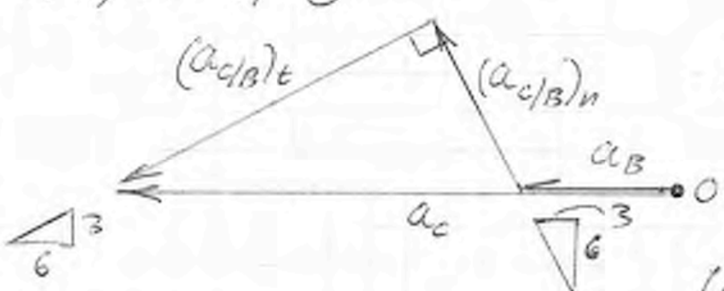
$$= \underline{a}_B + (\underline{a}_{c/B})_t + (\underline{a}_{c/B})_n$$

$$a_B = \omega_{AB}^2 \overline{AB} = (10)^2(0.6) = 60 \text{ m/s}^2$$

Using ICR of BC: $\omega_{BC} = \frac{v_B}{0.3} = \frac{\omega_{AB}\overline{AB}}{0.3} = \frac{(10)(0.6)}{0.3} = 20$ ↻

$$(a_{c/B})_n = \omega_{BC}^2 \overline{BC} = (20)^2\sqrt{0.3^2 + 0.6^2} = 268.3 \frac{m}{s^2}$$

Vector diagram of (1):



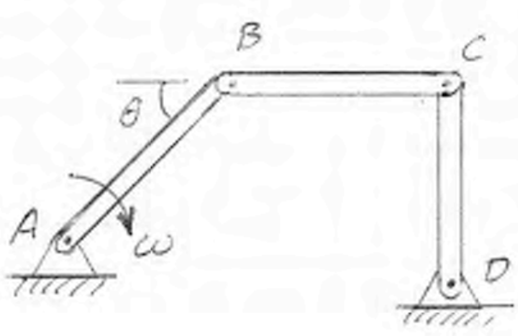
$$(a_{c/B})_t = (a_{c/B})_n \frac{6}{3}$$

$$= 536.7 \frac{m}{s^2}$$

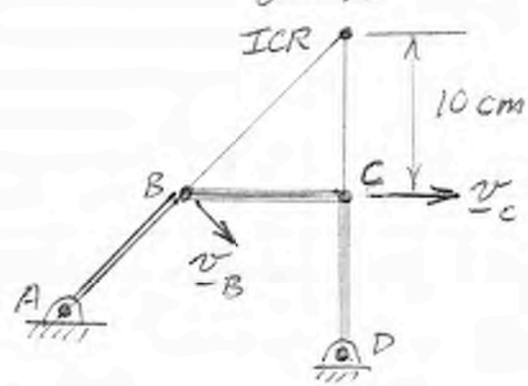
$$(a_{c/B})_t = \alpha_{BC} \overline{BC}$$

$$\alpha_{BC} = \frac{536.7}{\sqrt{0.45}} = 800 \frac{rad}{s^2}$$ ↻ ← Ans.

15.2



$\theta = 45°$



← Ans.

15.3

See bottom of 9 $\quad \underline{\omega}_{AB} = 5 \text{ rad/s}$

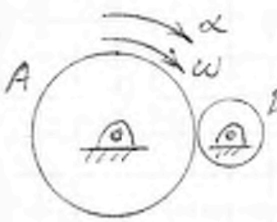
Using ICR of BC: $\quad \omega_{BC} = \dfrac{v_B}{\sqrt{2}\,0.1}$

$$\omega_{BC} = \frac{\omega_{AB}\,\overline{AB}}{\sqrt{2}\,0.1}$$

$$v_C = \frac{\omega_{AB}\,\overline{AB}}{\sqrt{2}\,\cancel{0.1}}\,\cancel{0.1}$$

$$\omega_{CD} = \frac{\omega_{AB}}{\sqrt{2}}\,\frac{\overline{AB}}{\overline{CD}} = \frac{(5)(0.14)}{\sqrt{2}\,(0.20)} = 2.47$$ ← Ans.

15.4



Gear A: $d_A = 20\text{ cm}$

Gear B: $d_B = 5\text{ cm}$

$\omega_A = 20 \text{ rad/s}$

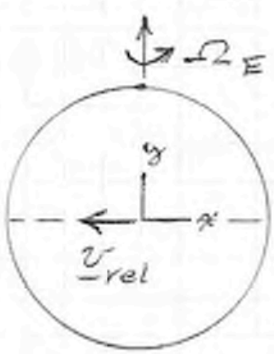
$\alpha_A = 4 \text{ rad/s}^2$

At point of engagement: $\alpha_A r_A = \alpha_B r_B$

$$\alpha_B = \alpha_A \frac{r_A}{r_B} = 4\left(\frac{10}{2.5}\right)$$

$$\alpha_B = 16 \ \frac{\text{rad}}{\text{s}^2}$$ ← Ans.

15.5

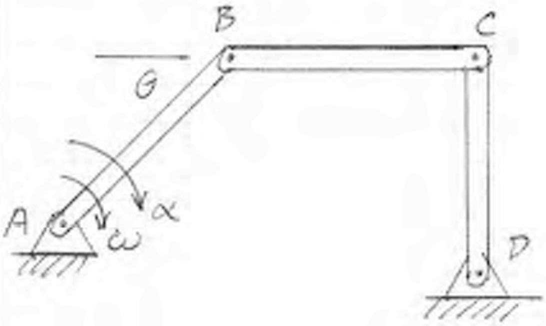


$$2\,\underline{\Omega}_E \times \underline{v}_{rel} = 2\,\Omega_E\,\underline{j} \times (-v_{rel}\,\underline{i})$$

$$= 2\,\Omega_E\, v_{rel}\,\underbrace{(\underline{j} \times -\underline{i})}_{\underline{k}}$$

$$= 2\,\Omega_E\, v_{rel}\,\underline{k}$$ ← Ans.

15.6

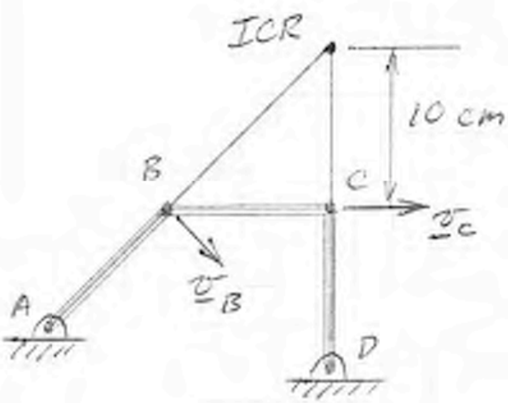


$\theta = 45°$

$\underline{\omega}_{AB} = 5\ \frac{rad}{s}$

$\underline{\alpha}_{AB} = 10\ \frac{rad}{s^2}$

Using the ICR of BC:



$$\omega_{BC} = \frac{v_B}{\sqrt{2}\, 0.1}$$

$$\omega_{BC} = \frac{\omega_{AB}\, \overline{AB}}{\sqrt{2}\, 0.1} = \frac{(5)(.1414)}{\sqrt{2}\ 0.1}$$

$$= 5\ \frac{rad}{s}$$

$$v_c = \omega_{BC}\, 0.1 = 0.5$$

$$v_c = \omega_{CD}\, \overline{CD}$$

$$\omega_{CD} = \frac{0.5}{0.2} = 2.5\ \frac{rad}{s}$$

$$\underline{a}_c = \underline{a}_B + \underline{a}_{c/B}$$

$$(\underline{a}_c)_t + (\underline{a}_c)_n = (\underline{a}_B)_t + (\underline{a}_B)_n + (\underline{a}_{c/B})_t + (\underline{a}_{c/B})_n \quad (1)$$

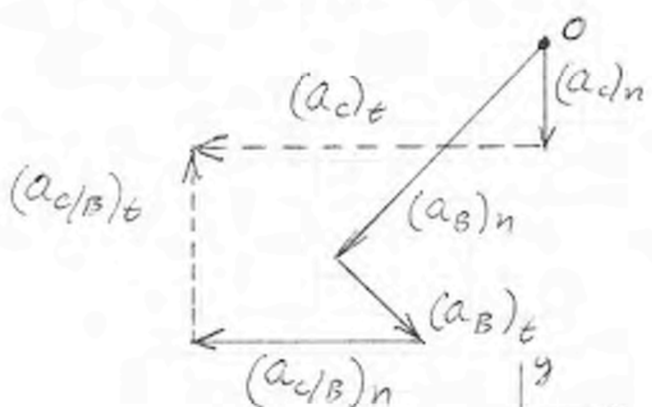
$$(a_c)_n = \omega_{CD}^2\, \overline{CD} = (2.5)^2(0.20) = 1.25\ m/s^2$$

$$(a_B)_t = \alpha_{AB}\, \overline{AB} = (10)(0.1414) = 1.414\ m/s^2$$

$$(a_B)_n = \omega_{AB}^2\, \overline{AB} = (5)^2(0.1414) = 3.535\ m/s^2$$

$$(a_{c/B})_n = \omega_{BC}^2\, \overline{BC} = (5)^2(0.1) = 2.5\ m/s^2$$

Vector diagram of (1):



$$(a_c)_t = \frac{(a_B)_n}{\sqrt{2}} - \frac{(a_B)_t}{\sqrt{2}} + (a_{c/B})_n$$

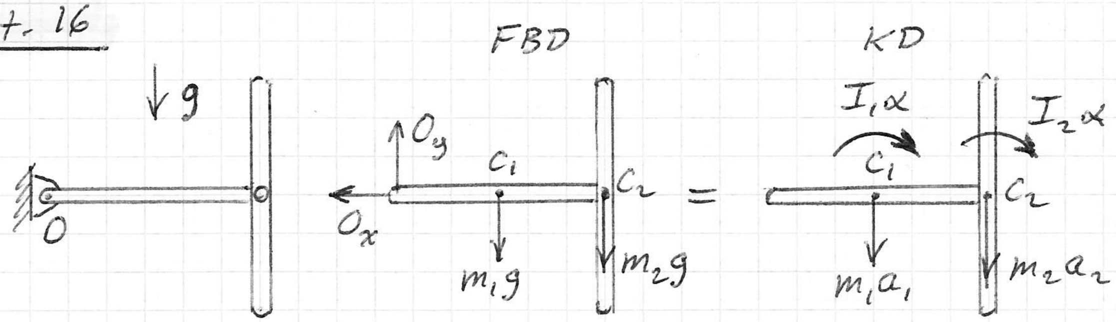
$$= \frac{3.535}{\sqrt{2}} - \frac{1.414}{\sqrt{2}} + 2.5$$

$$= 4\ m/s^2$$

$$\underline{a}_c = -4\underline{i} - 1.25\underline{j}\ ,\ \frac{m}{s^2}$$

Ans.

16.1



$$\curvearrowright + \sum M_O = m_1 a_1 \overline{OC_1} + I_1\alpha + m_2 a_2 \overline{OC_2} + I_2\alpha$$

$$a_1 = \overline{OC_1}\alpha, \quad a_2 = \overline{OC_2}\alpha$$

$$m_1 g \overline{OC_1} + m_2 g \overline{OC_2} = I_O \alpha$$

$$(20)(0.5)+(20)(1) = \left[\frac{1}{3}\left(\frac{20}{9.81}\right)1^2 + \frac{1}{12}\left(\frac{20}{9.81}\right)1^2 + \frac{20\times 1^2}{9.81}\right]\alpha$$

$$30 = 2.89\alpha$$

$$\alpha = 10.39 \text{ rad/s}^2 \curvearrowright$$

$$\sum F_y = -ma_1 - ma_2$$

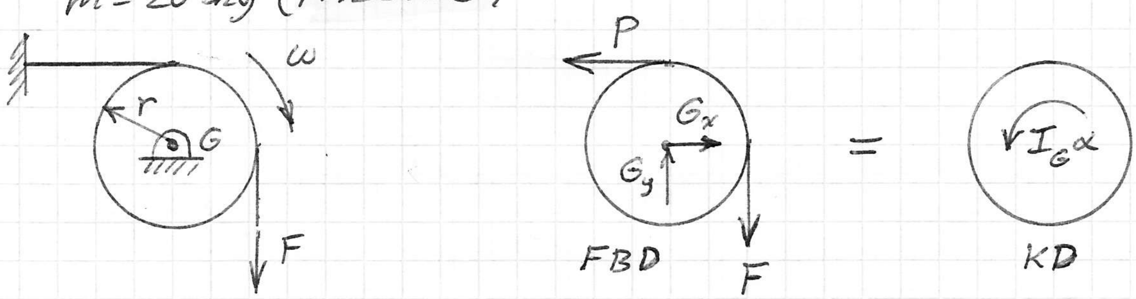
$$O_y - m_1 g - m_2 g = -ma_1 - ma_2$$

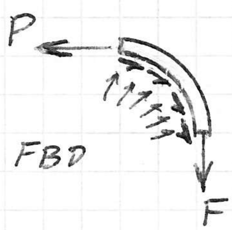
$$O_y = 20+20-\frac{20}{9.81}(0.5\times 10.39 + 1\times 10.39)$$

$$= 8.24 \text{ N} \longleftarrow \text{Ans.}$$

16.2

m = 20 kg (MISSING)





$$+\circlearrowleft \sum M_G = I_G\alpha$$

$$(P-F)r = \frac{1}{2}mr^2\alpha$$

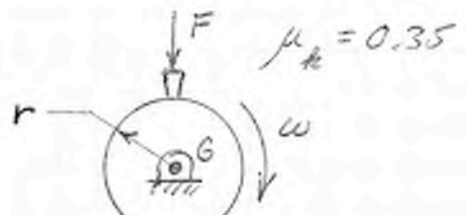
$$P = Fe^{\mu_k \pi/2}$$ (p. 73; Handbook)

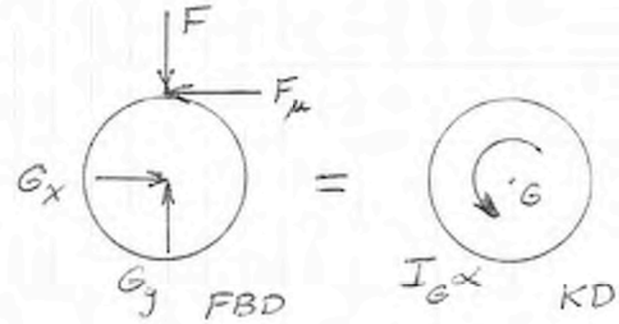
$$\alpha = \frac{2F(e^{\mu_k\pi/2}-1)}{mr} = \frac{2(350)(e^{0.35\pi/2}-1)}{(20)(0.25)}$$

$$= 102.6 \text{ rad/s}^2$$

$$\omega = \omega_0 - \alpha t, \quad \omega = 0 \text{ at } t = \frac{\omega_0}{\alpha} = \frac{500\times 2\pi}{60}\frac{1}{102.6} = 0.510 \text{ s} \longleftarrow \text{Ans.}$$

16.3





$$+\circlearrowleft \sum M_G = I_G \alpha$$

$$F_\mu r = I_G \alpha \quad (1)$$

$$F_\mu = \mu_k F \quad (2)$$

(2) → (1) :

$$\alpha = \frac{\mu_k F r}{I_G} \quad (3)$$

$\omega d\omega = \alpha d\theta \Longrightarrow \omega^2 = \omega_0^2 - 2\alpha\theta$, $\omega = 0$ when $\theta = \frac{\pi}{2}$:

$$\alpha = \frac{\omega_0^2}{2\theta} = \left(\frac{60 \times 2\pi}{60}\right)^2 \frac{1}{\pi}$$

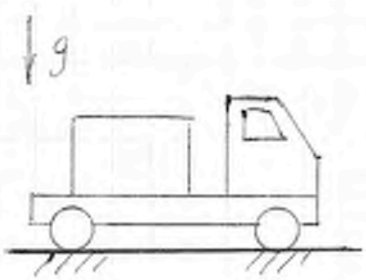
$$= 4\pi \quad (4)$$

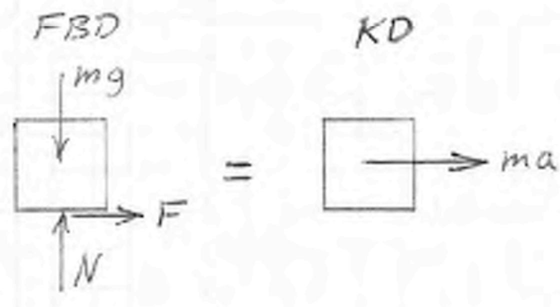
(4) → (3)

$$F = \frac{I_G \alpha}{\mu_k r} = \frac{\frac{1}{2}(5)(0.35)^2 4\pi}{(0.35)(0.35)}$$

$$= 31.4 N$$ ← Ans.

16.4





$$\sum F_y = 0 \Rightarrow N = mg \quad (1)$$

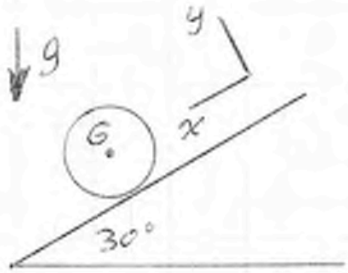
$$F = ma \quad (2)$$

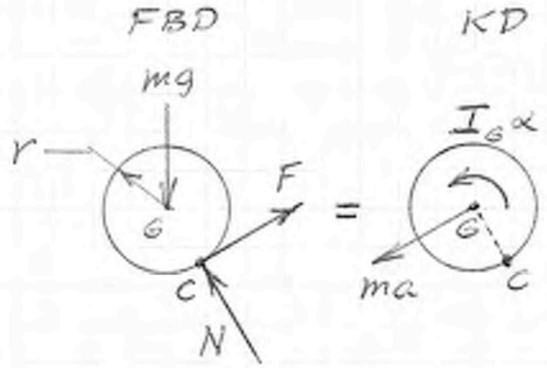
$$F = \mu_s N \quad (3)$$

From (1), (2) & (3) : $a = \frac{1}{m}\mu_s mg = 0.4(9.81)$

$$= 3.92 \frac{m}{s^2}$$ ← Ans.

16.5





$a = r\alpha$

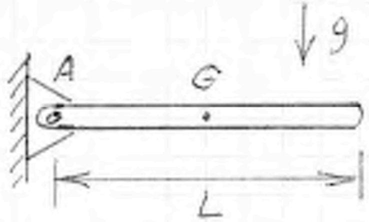
$$\overset{+}{\circlearrowleft}\ \Sigma M_C = mar + I_G\alpha$$

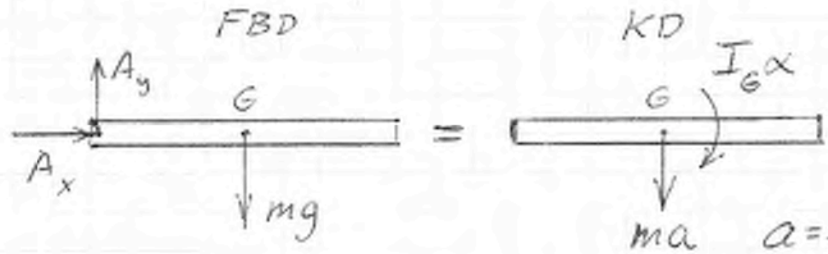
$$mgr\sin 30^\circ = mar + \frac{1}{2}mr^2\frac{a}{r}$$

$$a = \frac{2g\sin 30^\circ}{3} = \frac{2(9.81)\sin 30^\circ}{3}$$

$$a = 3.27\ m/s^2 \longleftarrow \text{Ans.}$$

16.6





$a = \frac{L}{2}\alpha$

$$\Sigma F_y = -ma \Rightarrow A_y - mg = -ma \quad (1)$$

$$\overset{+}{\circlearrowright}\ \Sigma M_A = ma\frac{L}{2} + I_G\alpha$$

$$mg\frac{L}{2} = ma\frac{L}{2} + \frac{1}{12}mL^2\frac{2a}{L}$$

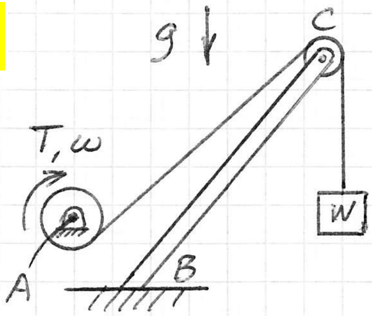
$$mg\frac{L}{2} = \frac{2}{3}maL \Rightarrow a = \frac{3}{4}g \quad (2)$$

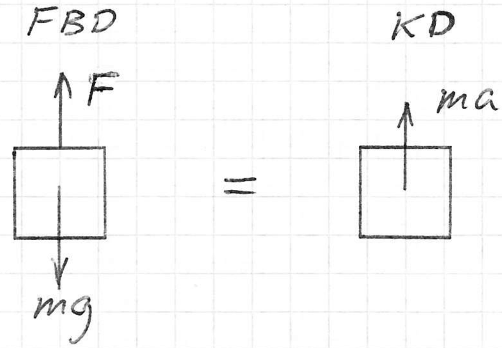
$$(2) \rightarrow (1):\ A_y = mg - ma = mg - \frac{3}{4}mg = \frac{mg}{4}$$

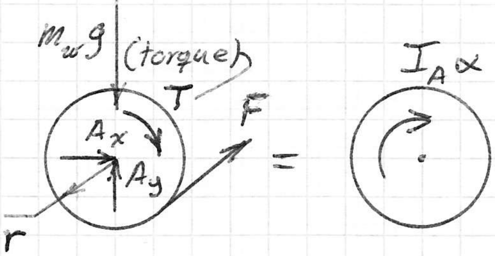
$$= 10/4 = 2.5\ N \longleftarrow \text{Ans.}$$

Chpt. 17

17.1







$r\alpha = a$

$$\Sigma F_y = ma$$

$$F - mg = ma$$

$$F = ma + mg$$

$$= \frac{10{,}000}{9.81}(1) + 10{,}000 = 11{,}019 \text{ N}$$

$$\circlearrowleft + \ \Sigma M_A = I_A \alpha$$

$$T - Fr = I_A \alpha$$

$$T = Fr + I_A a/r$$

radius of gyration $= \sqrt{\frac{I_A}{m_w}}$

$$= (11{,}019)(0.5) + (0.4)^2(600)(1)/0.5$$

$$= 5702 \text{ N}\cdot\text{m}$$

$$P = \text{Power} = T\omega = T\frac{v}{r} = 5702\,\frac{10/60}{0.5}$$

$$P = 1.90 \text{ kW}$$

← Ans.

17.2

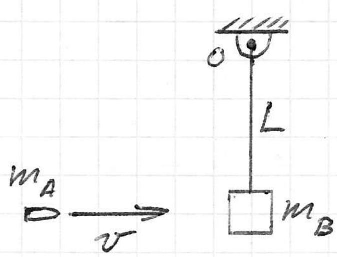
Above figure

$$P = Fv \qquad F = mg$$

$$= mgv = (10{,}000)\left(\frac{10}{60}\right) = 1.67 \text{ kW}$$

← Ans

17.3



Conservation of angular momentum about O:

$$m_A v \cancel{L} = (m_A + m_B)(\omega L)\cancel{L}$$

$$(0.035)(300) = (500.035)(0.5\omega)$$

$$\omega = 0.0420 \text{ rad/s}$$

← Ans

17.4

Above figure.

Conservation of energy after impact:

$$0 = \Delta T + \Delta V_g \Rightarrow \frac{1}{2}(m_A + m_B)(\omega L)^2 = (m_A + m_B)gL \times (1 - \cos 30^\circ)$$

$$\omega^2 = \frac{(2)(9.81)(1 - \cos 30^\circ)}{0.5}$$

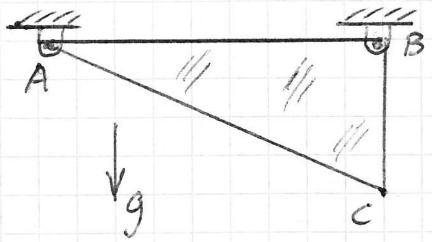
$$\omega = 2.29 \text{ rad/s}$$

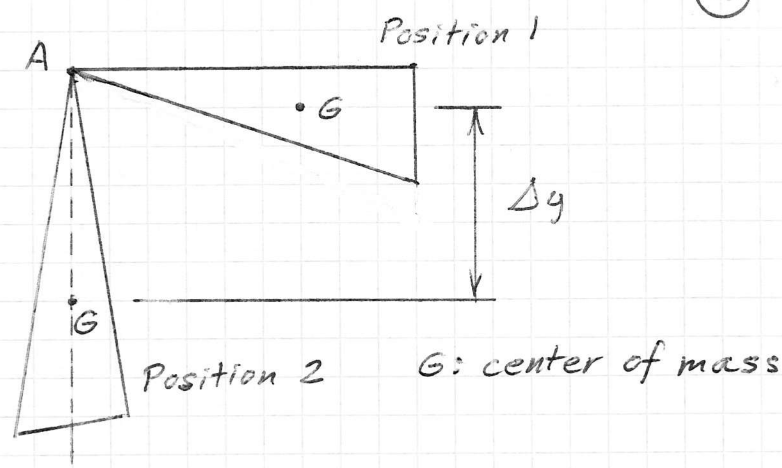
Conser. of momentum during impact

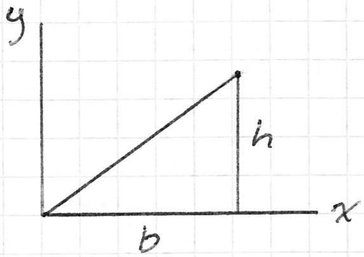
$$(0.035)v = (500.035)(0.5\omega) \Rightarrow v = 16{,}379 \text{ m/s}$$

← Ans.

17.5







Page 74, Handbook:

$$I_x = bh^3/12, \quad I_y = \frac{b^3h}{4}$$

Therefore, $I_A = (I_x + I_y)\rho t$; ρ = density, t = thickness

$$= \frac{bh\rho t}{2}\left(\frac{h^2}{6} + \frac{b^2}{2}\right)$$

$$m = bh\rho t/2$$

$$I_A = m\left(\frac{h^2}{6} + \frac{b^2}{2}\right)$$

$$0 = \Delta T + \Delta V_g \Longrightarrow \frac{1}{2} I_A \omega^2 = mg\,\Delta y$$

$$\frac{1}{2}\cancel{m}\left(\frac{h^2}{6} + \frac{b^2}{2}\right)\omega^2 = \cancel{m}g\left\{\left[\left(\frac{2}{3}b\right)^2 + \left(\frac{h}{3}\right)^2\right]^{1/2} - \frac{h}{3}\right\}$$

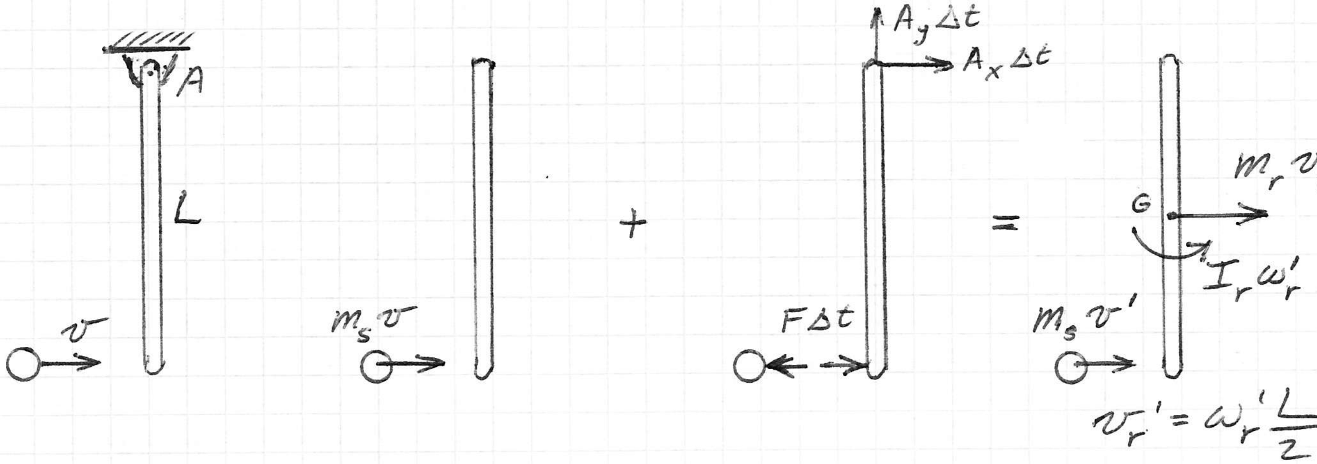
$$\frac{1}{2}\left(\frac{0.3^2}{6} + \frac{0.9^2}{2}\right)\omega^2 = 9.81\left\{\left[\left(\frac{2}{3}0.9\right)^2 + \left(\frac{0.3}{3}\right)^2\right]^{1/2} - \frac{0.3}{3}\right\}$$

$$0.21\,\omega^2 = 4.99$$

$$\omega^2 = 23.7$$

$$\omega = 4.87 \text{ rad/s} \longleftarrow \text{Ans.}$$

17.6



Conservation of angular momentum about A.

$$m_s v L = m_s v' L + I_r \omega_r' + m_r v_r' \frac{L}{2}$$

$$m_s v L = m_s v' L + \frac{1}{12} m_r L^2 \frac{2 v_r'}{L} + m_r v_r' \frac{L}{2}$$

$$m_s v \cancel{L} = m_s v' \cancel{L} + \frac{2}{3} m_r v_r' \cancel{L} \quad (1)$$

$$e = \frac{2 v_r' - v'}{v} \quad (2)$$

$$(1) \Longrightarrow (1)(10) = (1) v' + \frac{2}{3}(10) v_r' \quad (3)$$

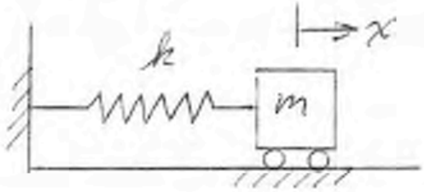
$$(2) \Longrightarrow (0.7)(10) = - v' + 2 v_r' \quad (4)$$

$$(3) - \frac{10}{3} \times (4) \qquad 10 - (7)\left(\frac{10}{3}\right) = \left(1 + \frac{10}{3}\right) v'$$

$$v' = -3.08 \ \frac{m}{s} \longleftarrow \text{Ans.}$$

Chpt. 19

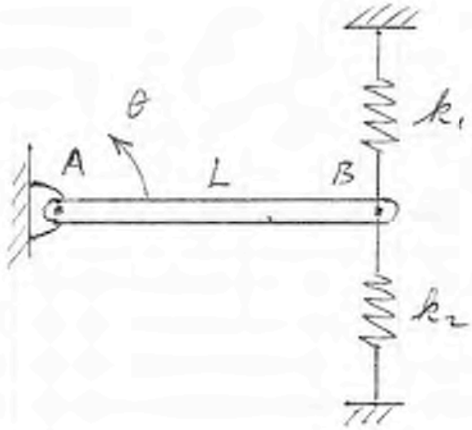
19.1

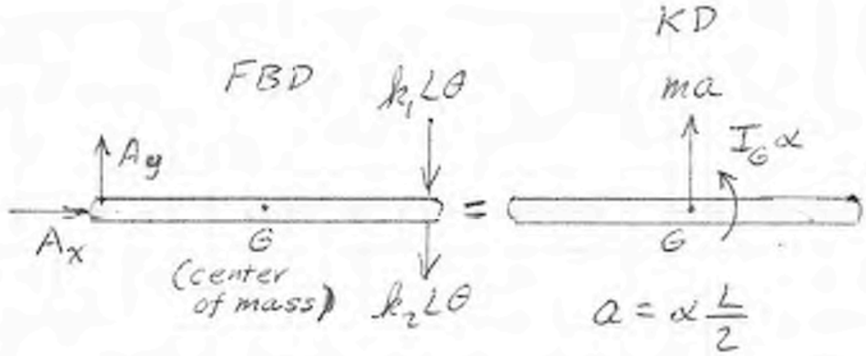


$k = 800\ N/m$
$m = 6\ kg$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{800}{6}} = 11.55\ \frac{rad}{s} \longleftarrow \text{Ans.}$$

19.2





$$a = \alpha \frac{L}{2}$$

$$\circlearrowleft \Sigma M_A = ma\frac{L}{2} + I_G \alpha$$

$$-(k_1 + k_2)L^2\theta = m\alpha\left(\frac{L}{2}\right)^2 + I_G\alpha$$

$$= I_A \alpha$$

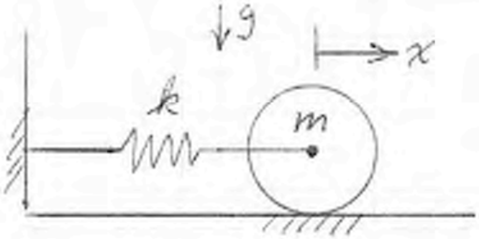
$$I_A \ddot{\theta} + (k_1 + k_2)L^2\theta = 0$$

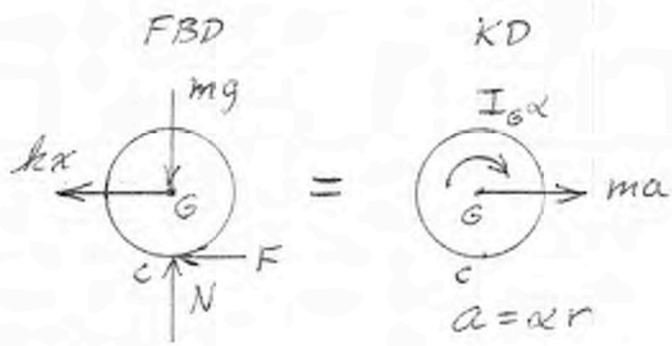
$$\omega_n = \sqrt{\frac{(k_1+k_2)L^2}{I_A}} = \sqrt{\frac{(k_1+k_2)L^2}{\frac{1}{3}mL^2}} = \sqrt{\frac{1600 \times 3}{10}}$$

$$= 21.9\ rad/s$$

$$\omega_n = \frac{2\pi}{\tau} \Longrightarrow \tau = 0.287\,s \longleftarrow \text{Ans.}$$

19.3





$$a = \alpha r$$

$$\circlearrowleft + \Sigma M_c = mar + I_G\alpha$$

$$-kxr = m\ddot{x}r + \frac{1}{2}mr^2\frac{\ddot{x}}{r}$$

(Cont'd next page)

$$\frac{3}{2}m\ddot{x} + kx = 0$$

$$\omega_n = \sqrt{\frac{k}{\frac{3}{2}m}} = \sqrt{\frac{850}{\frac{3}{2}10}} = 7.53 \ \frac{rad}{s}$$

$$x = 0.02\cos(\omega_n t)$$

$$\dot{x} = 0.02\,\omega_n(-\sin(\omega_n t))$$

$$|\dot{x}|_{max} = 0.02\,\omega_n = 0.151 \ \frac{m}{s} \leftarrow \text{Ans.}$$

19.4 Above figure $\omega_n = \frac{2\pi}{\tau} \Rightarrow \tau = 0.835\,s \leftarrow$ Ans.

19.5 Figure on top of page 18

$$x = 0.030\cos(\omega_n t)$$

$$\dot{x} = -0.03\,\omega_n \sin(\omega_n t)$$

$$|\dot{x}|_{max} = 0.03\,\omega_n = 0.346 \ m/s \leftarrow \text{Ans.}$$

19.6

$$\ddot{x} = -0.03\,\omega_n^2 \cos(\omega_n t)$$

$$|\ddot{x}|_{max} = 0.03\,\omega_n^2 = 4\ m/s^2 \leftarrow \text{Ans.}$$