Cal State Los Angeles Department of Mathematics
Complex Analysis Comprehensive Examination
Spring 2023
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Directions: Do five of the following seven problems. If you turn in more than five, the best five will be used.

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1. Let $f(z) = e^{\overline{z}}$. Show that $f'(z)$ does not exist for any complex number z.

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2. Let $S = \{x + iy \mid 0 \leq x \leq 2 \text{ and } 3\pi/4 < y \leq 5\pi/4\}$.

(a) Sketch S.

(b) What is the image of S under the function $f(z) = e^z$? Sketch it. Label some points on the graph so that it is accurately described.

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3. Calculate $\displaystyle\int_\gamma \frac{1}{(z^2 + z + 1)^2}\, dz$ where γ is the circle $|z| = 2$ oriented counterclockwise.

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4. Find all the singular points and residues of $f(z) = \dfrac{1}{e^z - 1}$.

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5. Show that $z^6 + 9z^4 + z^3 + 2z + 4$ has four roots inside the unit circle.

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6. Suppose that $f(z) = u(x, y) + iv(x, y)$ is a continuous function on a compact region R and $f(z)$ is analytic and non-constant in the interior of R. Show that the component function $u(x, y)$ attains a maximum value on the boundary of R and never in the interior of R.

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7. If possible, find an entire function $f(z)$ that maps the real and imaginary axes onto themselves, and such that $f(0) = 0$, $f(1) = 1$, $f(-1) = -1$, $f(i) = -i$, $f(-i) = i$. If no such function exists, then explain why it does not exist.
