Quiz 3, Math 209, Winter 2007 **Name:** Solutions

Directions: Show ALL of your work to get credit. If you leave something out, then you may be penalized. No calculators. Good luck!

1. [10 points] Let

$$f(x,y) = 2\sqrt{x} - y^2.$$

(a) Find the directional derivative of $f(x,y)$ at $P(1,5)$ in the direction of $Q(4,1)$.

(b) Find the maximum rate of change of $f(x,y)$ at $P(1,5)$ and the direction in which it occurs.

(a) $\vec{PQ} = \langle 4-1, 1-5 \rangle = \langle 3, -4 \rangle$

$\|\vec{PQ}\| = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

So, $\vec{u} = \frac{1}{5}\langle 3, -4 \rangle = \langle \frac{3}{5}, \frac{-4}{5} \rangle$

$D_{\vec{u}} f(1,5) = \nabla f(1,5) \cdot \vec{u} = \langle 1, -10 \rangle \cdot \langle \frac{3}{5}, \frac{-4}{5} \rangle$

$= \frac{3}{5} + \frac{40}{5} = \frac{43}{5}$

$f_x(x,y) = \frac{1}{\sqrt{x}}$

$f_y(x,y) = -2y$

(b) direction is $\nabla f(1,5) = \langle 1, -10 \rangle$

maximum rate of change is $\|\nabla f(1,5)\| =$

$= \|\langle 1, -10 \rangle\|$

$= \sqrt{1^2 + (-10)^2} = \sqrt{101}$

2. [10 points] Find the local maximum and minimum values, and saddle points of the function

$$f(x,y) = \frac{1}{3}x^3 - x + 3y^2$$

Recall that $D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - [f_{x\cancel{x}y}(a,b)]^2$.

$$\left.\begin{array}{l} f_x(x,y) = x^2 - 1 \\ f_y(x,y) = 6y \end{array}\right\} \quad \left.\begin{array}{l} x^2 - 1 = 0 \\ 6y = 0 \end{array}\right\} \quad \begin{array}{l} x = \pm 1 \\ y = 0 \end{array}$$

So, critical points are $(1,0)$ and $(-1,0)$.

$f_{xx}(x,y) = 2x$

$f_{yy}(x,y) = 6$

$f_{xy}(x,y) = 0$

At $(1,0)$, $D(1,0) = f_{xx}(1,0)f_{yy}(1,0) - [f_{xy}(1,0)]^2$

$= 2 \cdot 6 - 0^2 = 12 > 0$

and $f_{xx}(1,0) = 2 > 0$

So, $(1,0)$ is a local minimum (with value $f(1,0) = -2/3$)

At $(-1,0)$, $D(-1,0) = f_{xx}(-1,0)f_{yy}(-1,0) - [f_{xy}(-1,0)]^2$

$= (-2)(6) - 0^2 = -12 < 0$

So, $(-1,0)$ is a saddle point.